Lecture 9:
High electron mobility transistor
(HEMT)
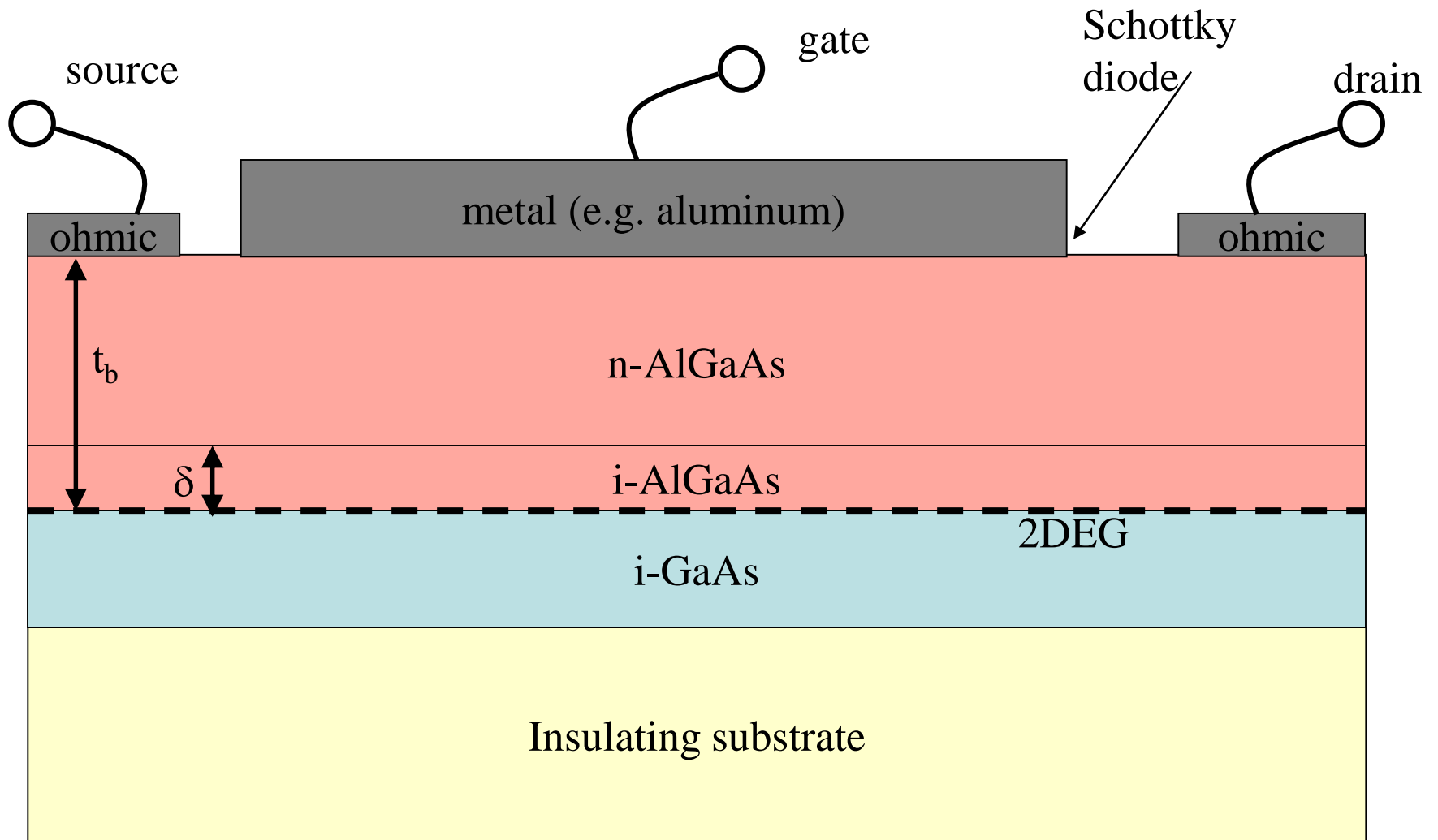
2DEG

E_0

E_0

ϕ_2

ϕ_1

E_c

E_c

ΔE_c

E_{Fermi}

GaAs

E_{Fermi}

AlGaAs

E_v

ΔE_v

E_v

HEMT:

source

gate

Schottky diode

drain

ohmic

metal (e.g. aluminum)

ohmic

t_b
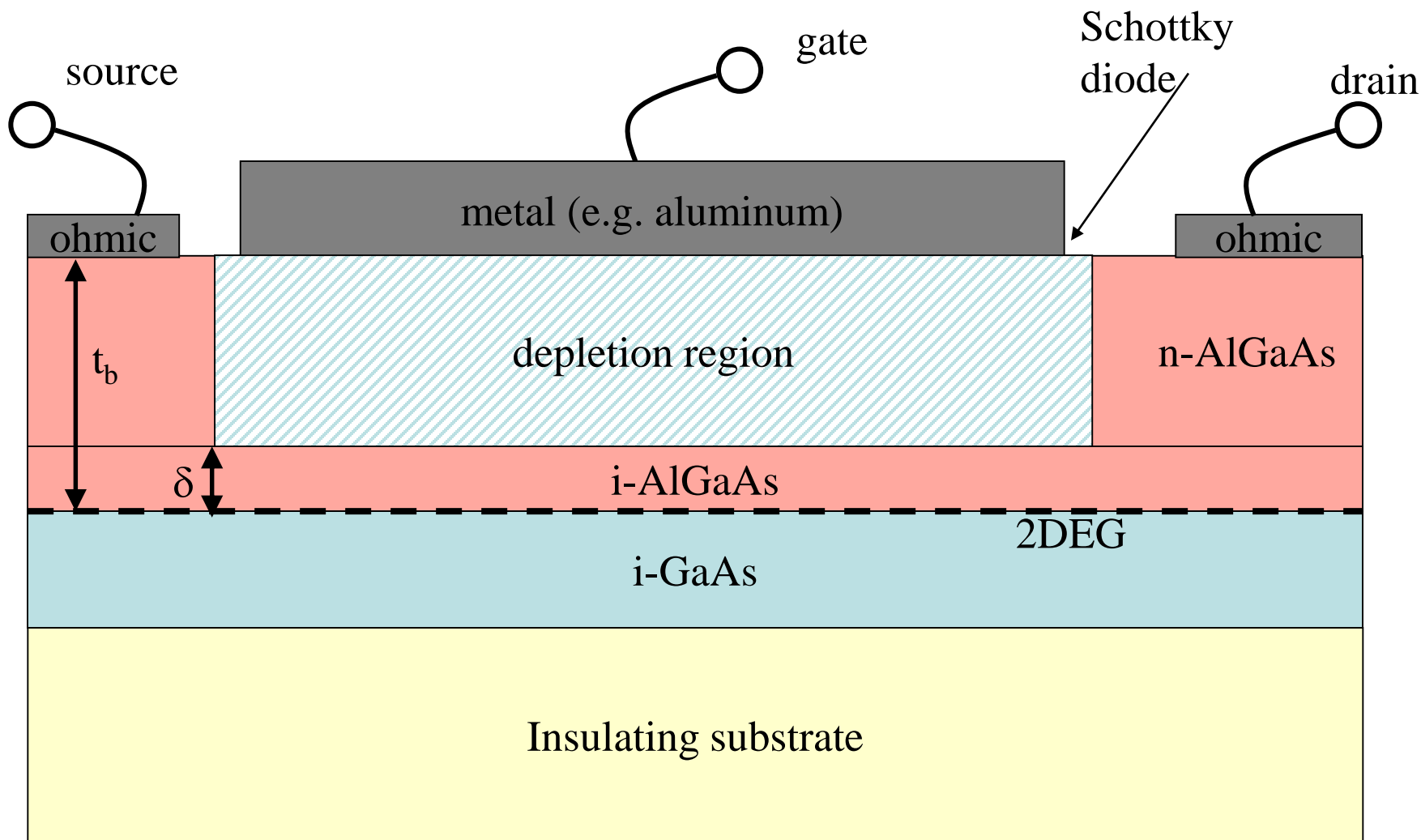
n-AlGaAs

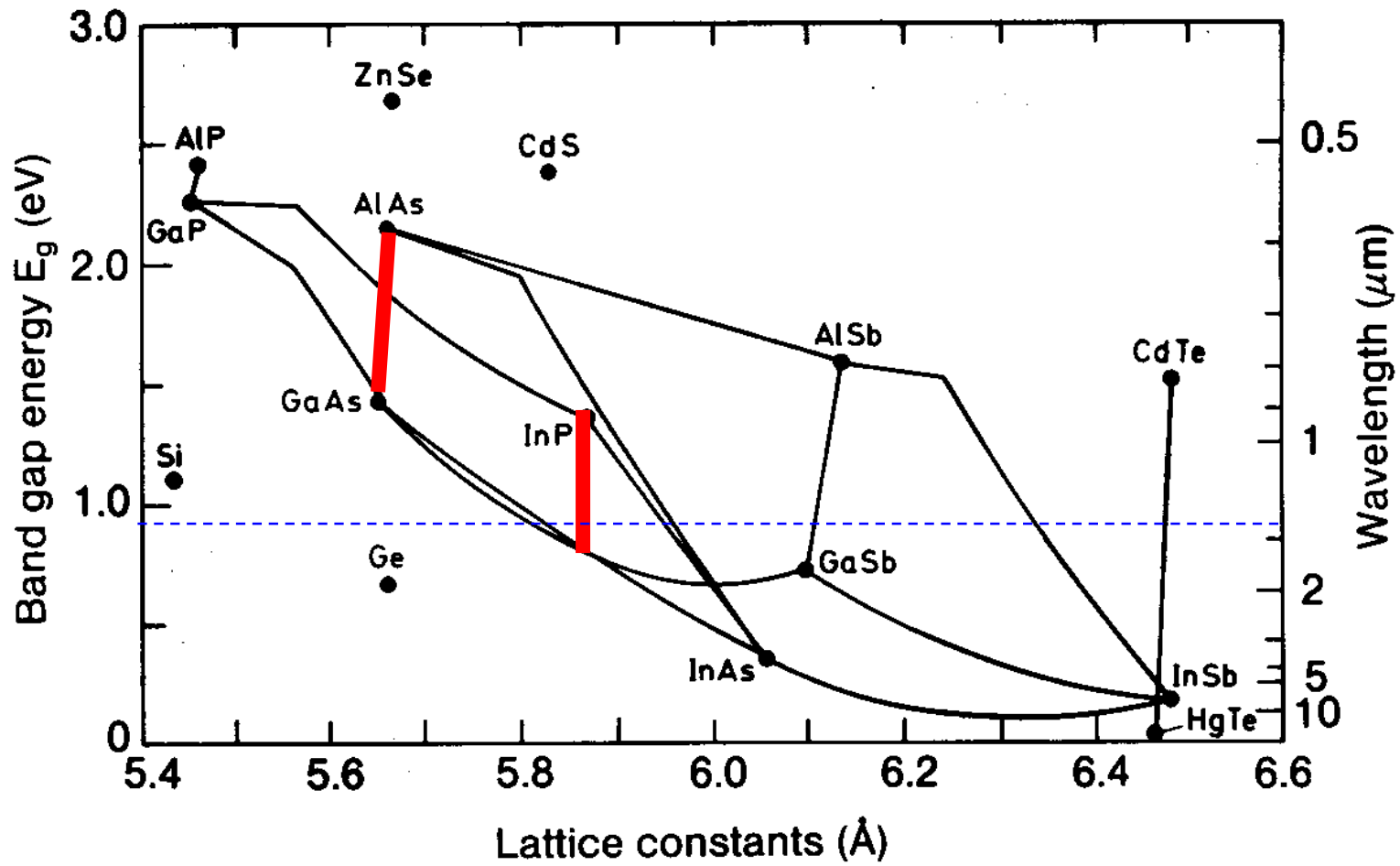δ

i-AlGaAs

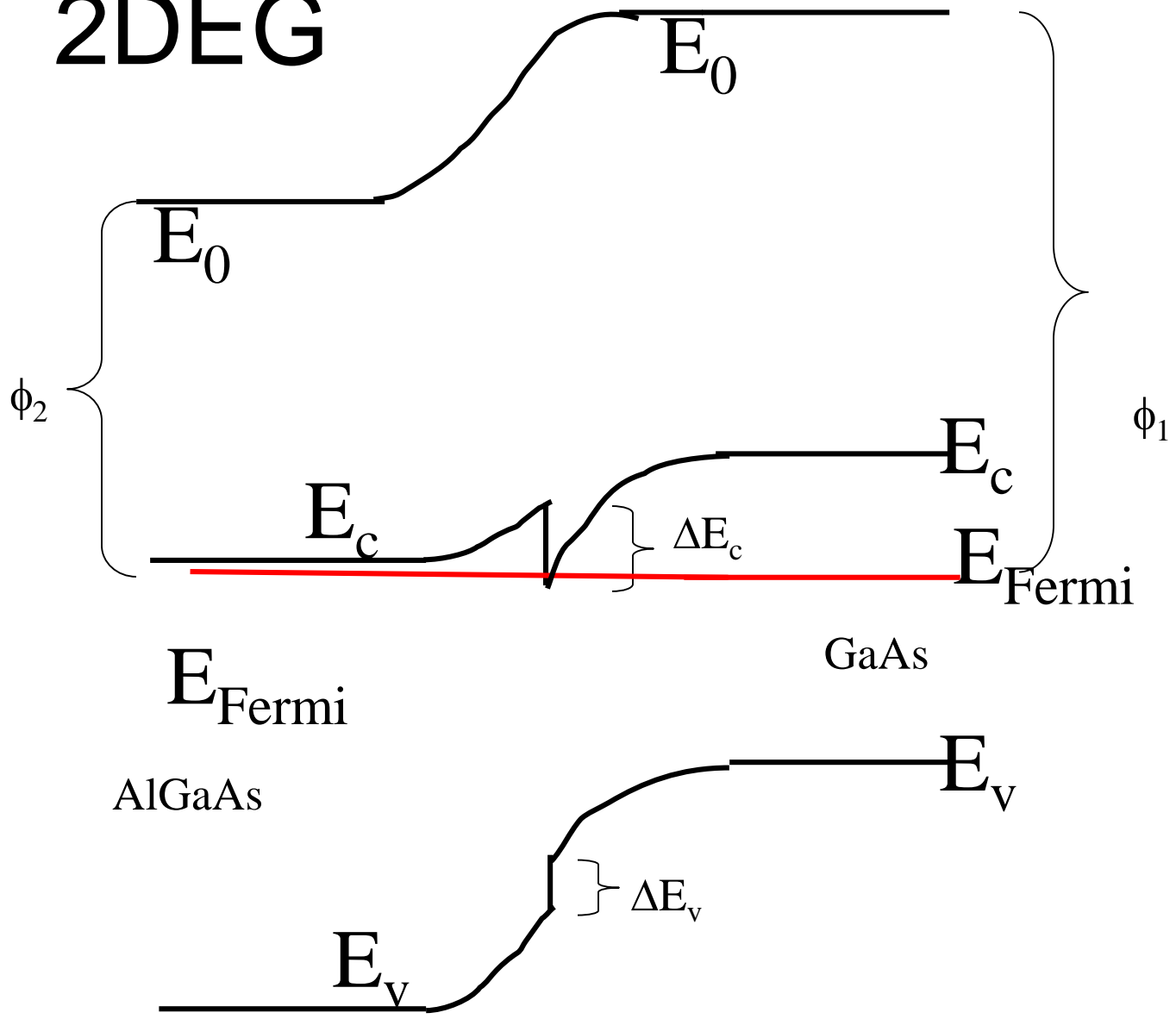2DEG

i-GaAs

Insulating substrate
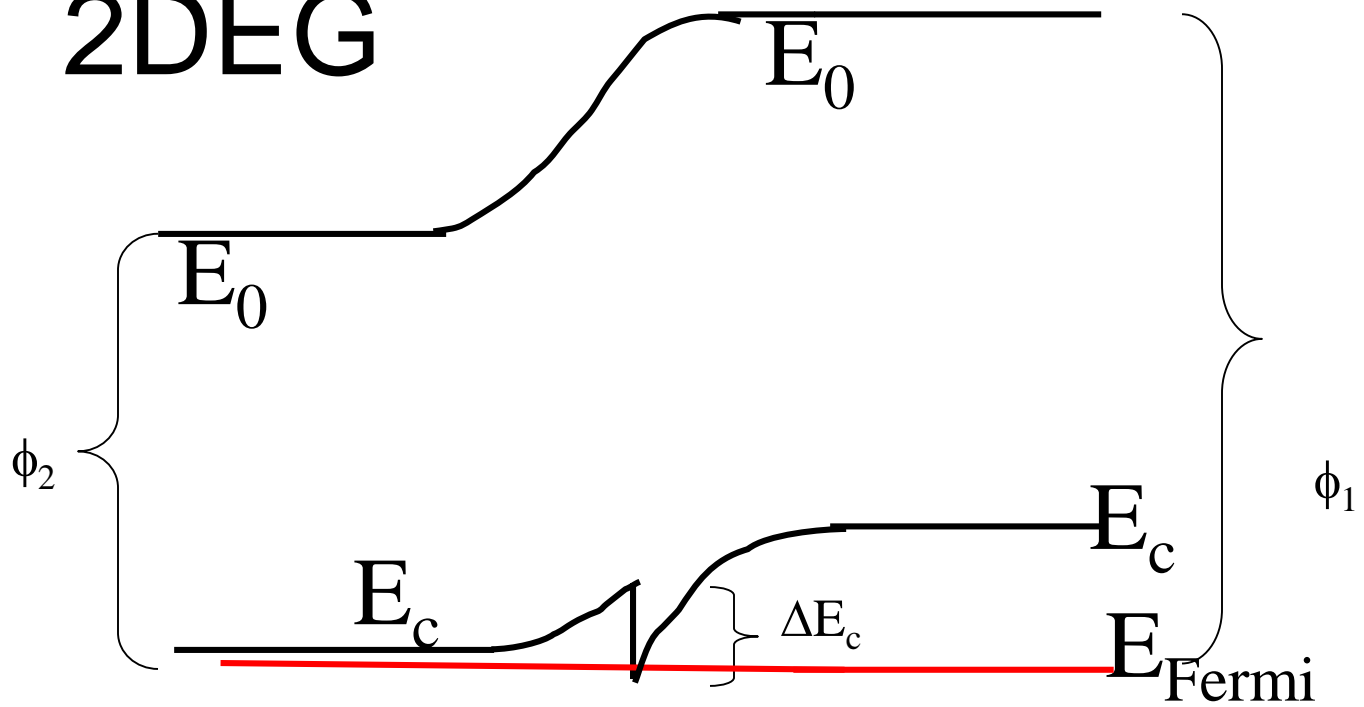
HEMT:

Many possible variations

- "Quantum well" is also popular.

- Highly doped material under ohmics for low contact resistance

- InP based materials

- GaN based materials
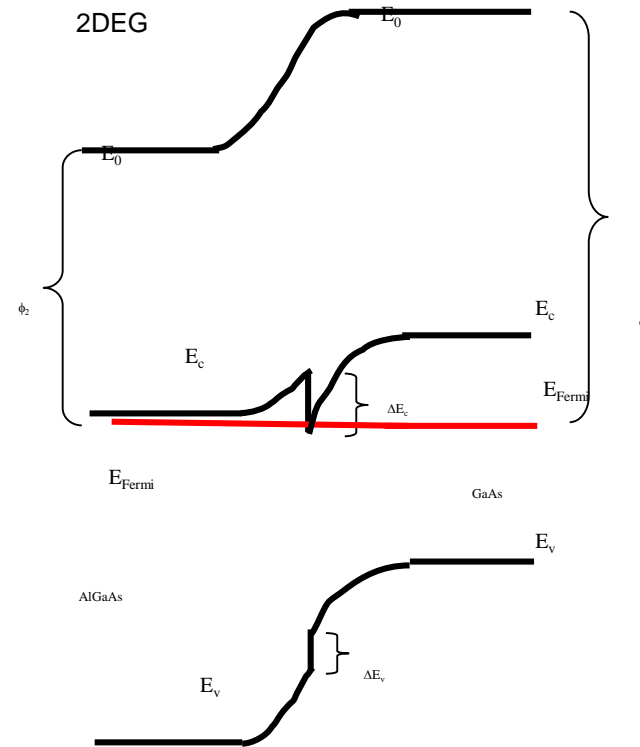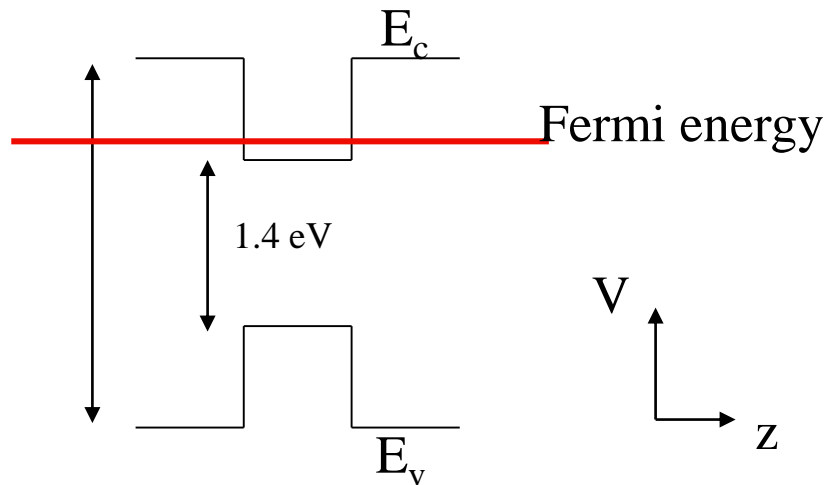
- (pHEMT: strained materials)

Band gaps

2DEG

E_0

E_0

ϕ_2

ϕ_1

E_c

E_c

ΔE_c

E_{Fermi}

E_{Fermi}

GaAs

AlGaAs

E_v

ΔE_v

E_v

Triangle vs. square well:

2DEG

E_c

Fermi energy

1.4 eV

E_v

V

z

L_0

L_0

ϕ_2

E_c

E_c

ΔE_c

E_{Fermi}

ϕ_1
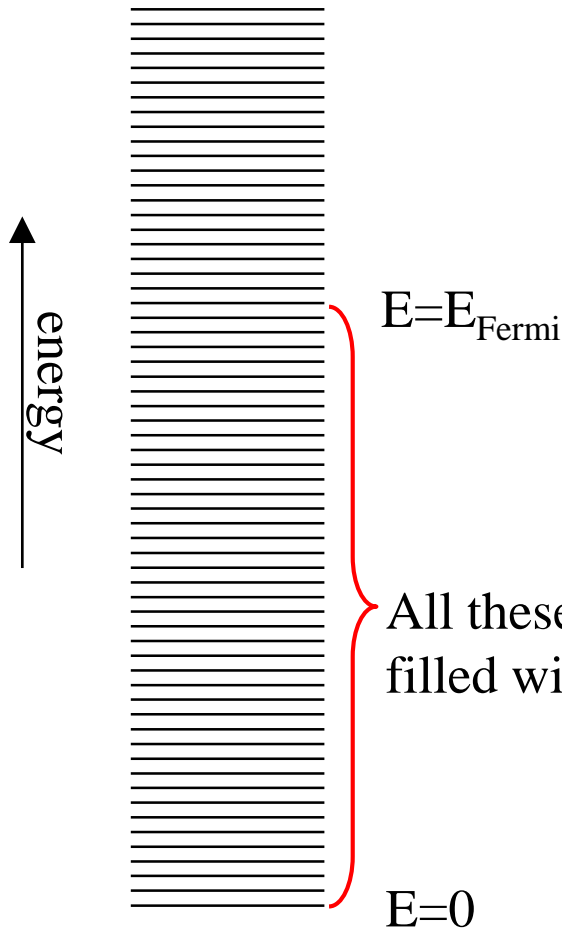
E_{Fermi}

GaAs

AlGaAs

E_v

E_v

ΔE_v

(Draw both bound states on board.
In particular discuss figure 5.21 from Liu.)
Also discuss shallow vs. wide wells on board.
(Typically 100 angstroms works.)

Fermi energy in 2 dimensions

$$\text{\# electrons} = \int_0^{E_f} N_{\text{E}}\,dE = \int_0^{E_f} L^2 \frac{m}{\pi\hbar}\,dE$$

$$\text{\# electrons} = L^2 \frac{m}{\pi\hbar} E_f$$

$$\Rightarrow E_f = \frac{\hbar\pi}{m}\left(\frac{\text{\# electrons}}{L^2}\right) = \frac{\hbar\pi}{m} n$$

energy

E=E$_{\text{Fermi}}$

All these states are filled with electrons.

E=0

In GaAs, 10^{11}cm^{-2} gives
E$_f$ ~ 1-10 meV
But 10^{12}cm^{-2} gives more than first subband.

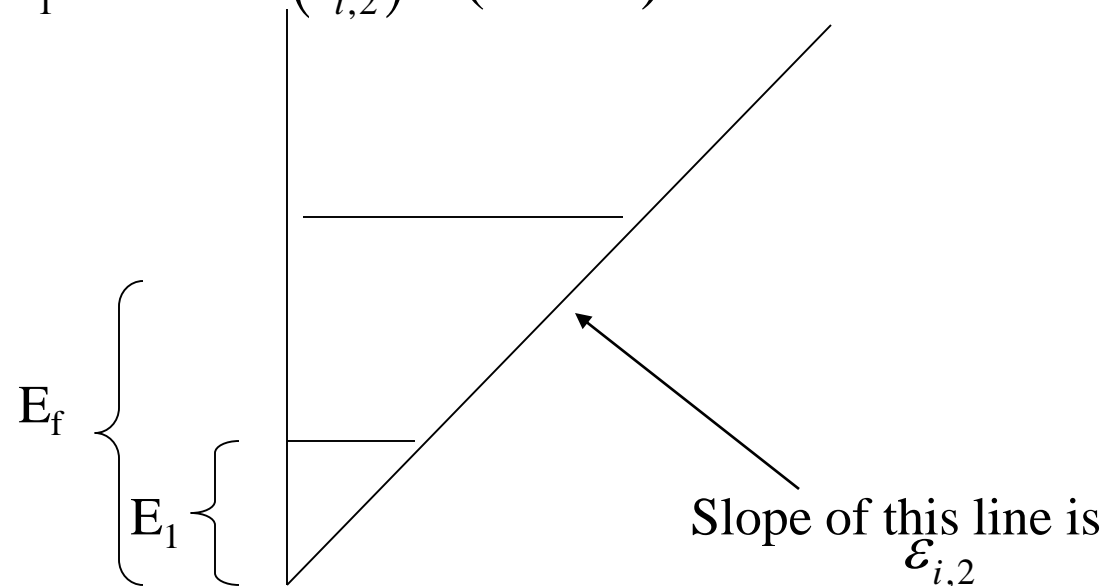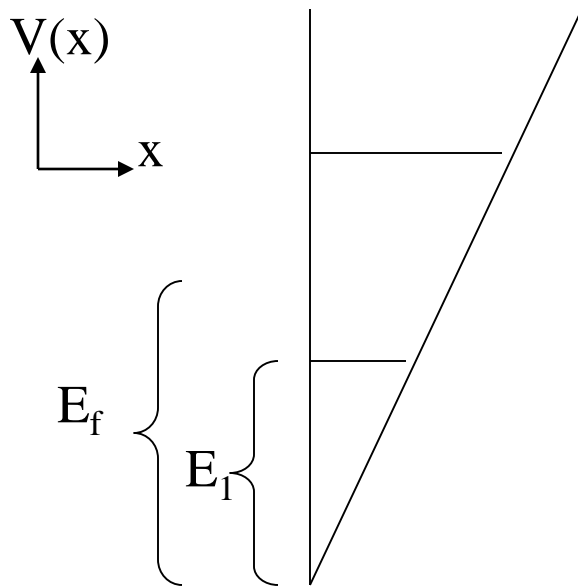Discuss "subband", how above integral gets modified.

Problem

- Presence of electrons changes shape of potential well.

- We need a way to account for this.

- Will do NOW.

- Why? We want to know how many electrons there are!

- Later, we want to know how gate voltage changes that.

Trianglular wells

From quantum mechanics:
$$E_n \approx \left(\frac{h^2}{8\pi^2 m^2}\right)^{1/3}\left(\frac{3\pi}{2}e\varepsilon_{i,2}\right)^{2/3}\left(n-\frac{1}{4}\right)^{2/3}$$

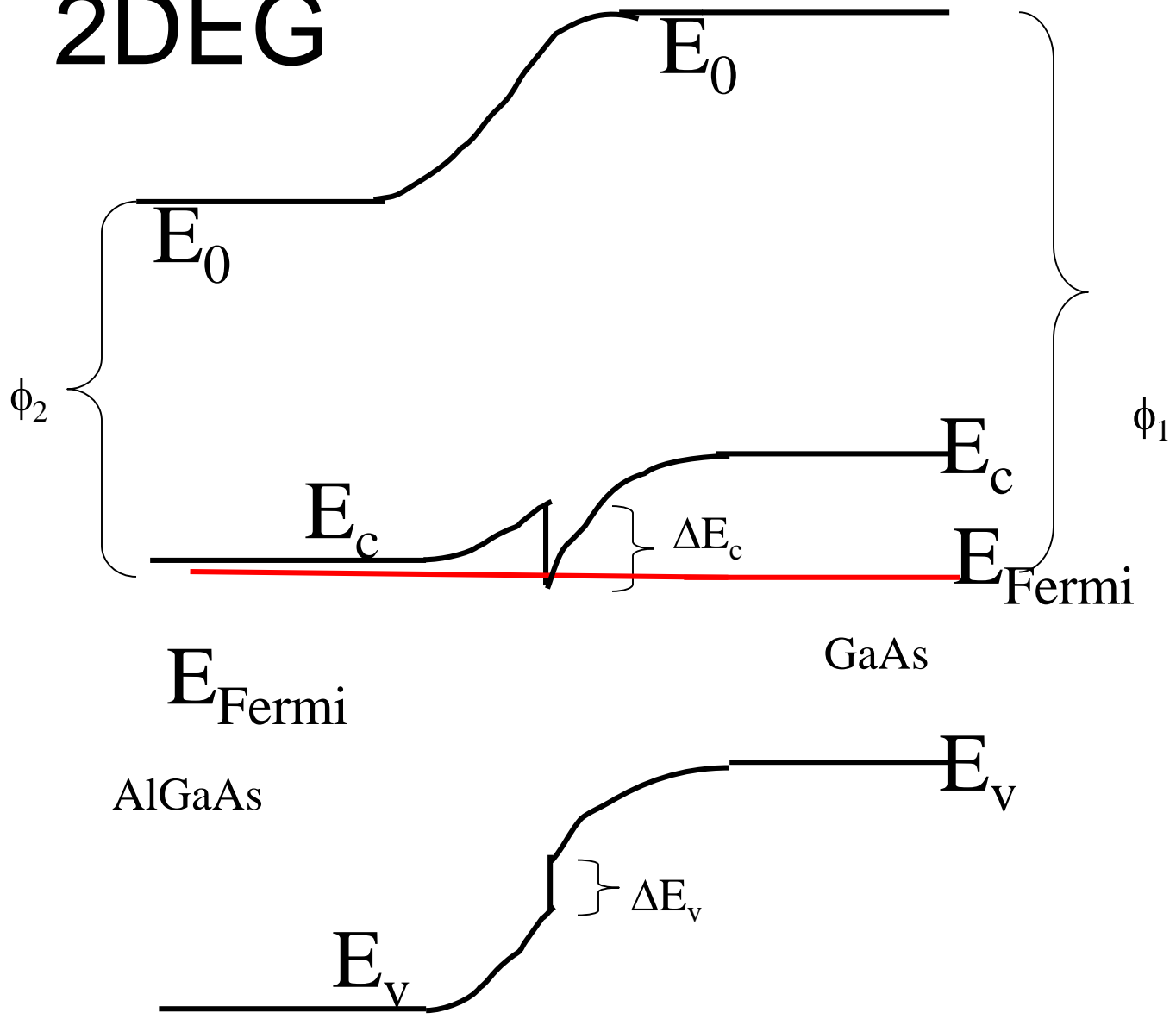$$E_1 \approx 4\cdot 10^{-5}\left(\varepsilon_{i,2}\right)^{2/3}(V/cm)$$

V(x)

x

E_f

E_1

E_f

E_1

Slope of this line is $\varepsilon_{i,2}$

$$E_f = \frac{\hbar^2\pi}{m}n_s \rightarrow \left(E_f - E_1\right)=\frac{\hbar\pi}{m}n_s$$

Note: Lecture 10 had error of hbar.

2DEG

E_0

E_0

ϕ_2

ϕ_1

E_c

E_c

ΔE_c

E_{Fermi}

E_{Fermi}

GaAs
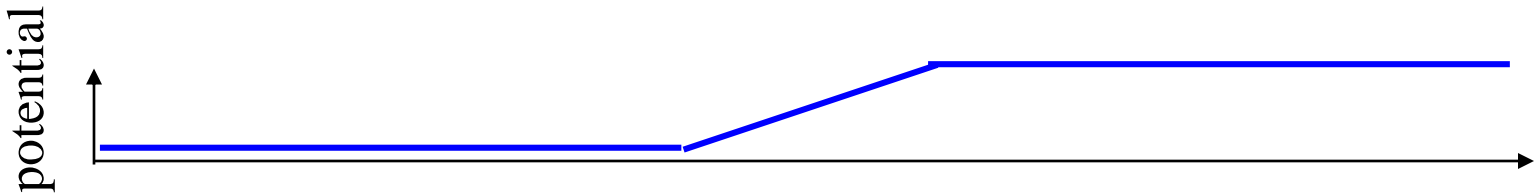
AlGaAs

E_v

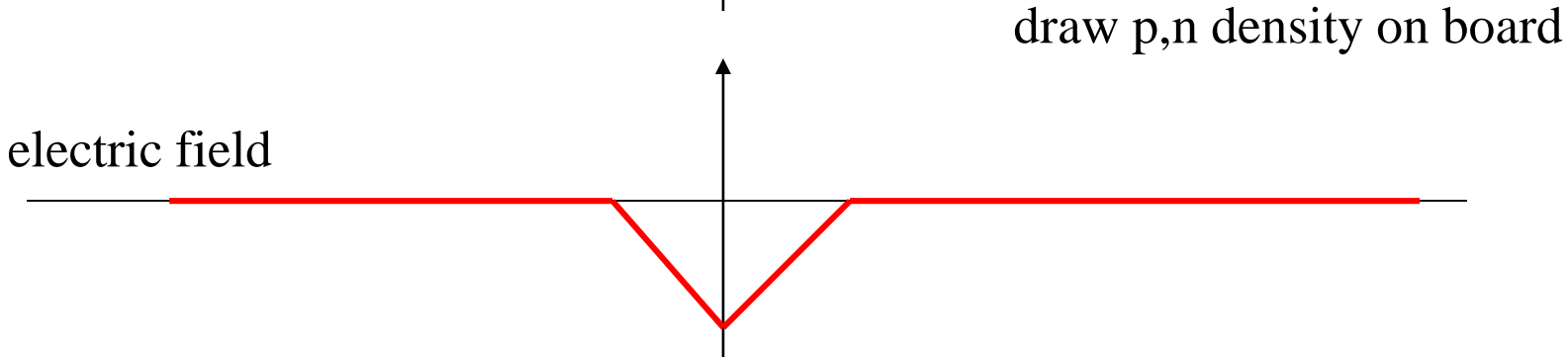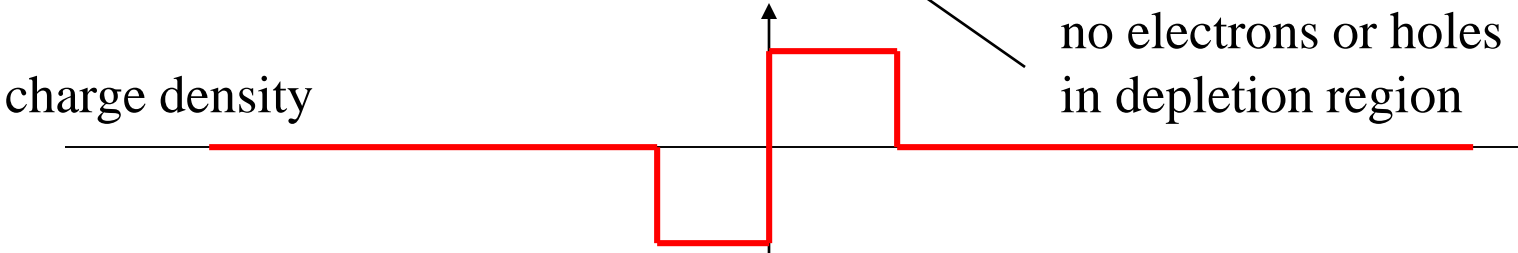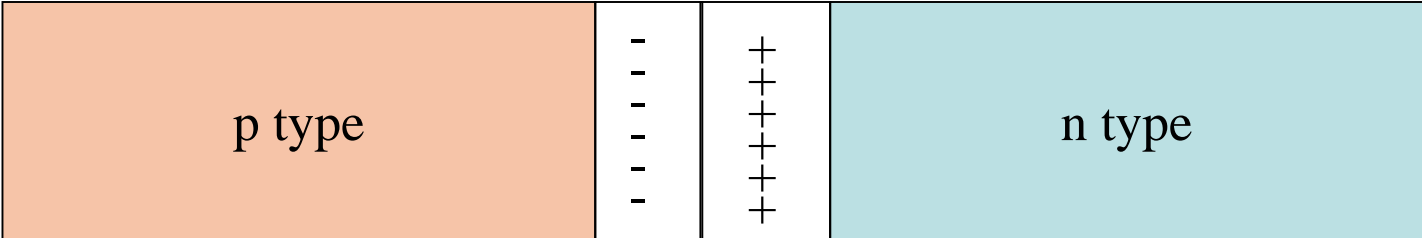ΔE_v

E_v

Poisson equation

Actually first of Maxwell's four equations:

$$\vec{\nabla} \cdot \vec{E} = -\frac{\rho}{\varepsilon}$$
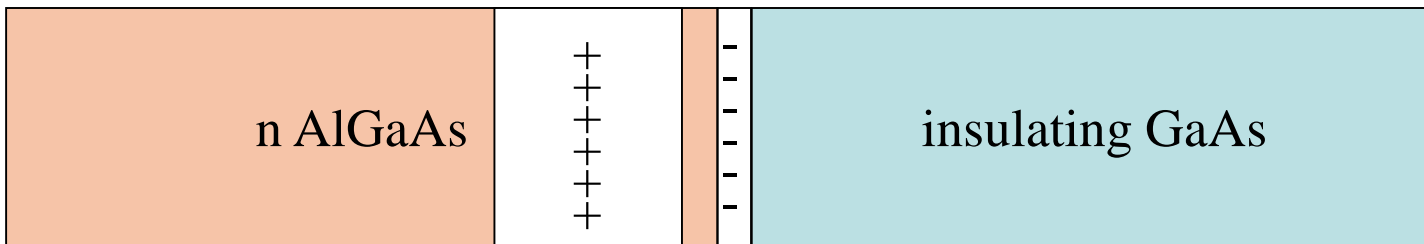
In the x-direction only:

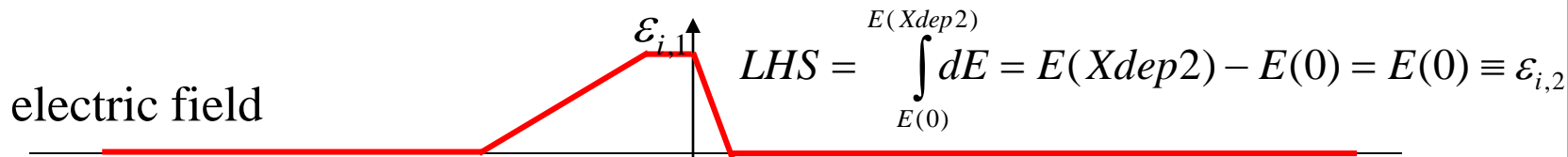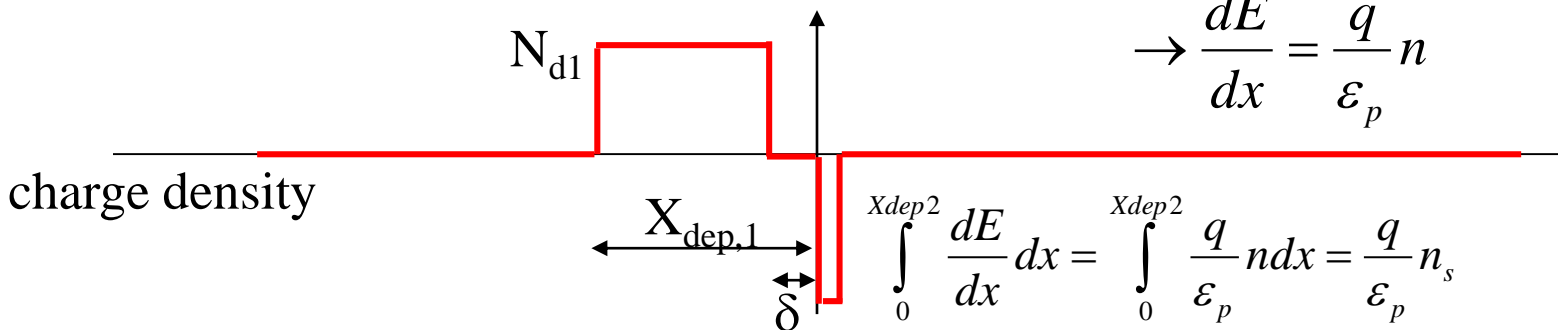$$\frac{dE}{dx} = \frac{q}{\varepsilon_p}\left(p - n + N_d^+ - N_a^-\right)$$

p n diode

p type

n type

no electrons or holes
in depletion region

charge density

draw p,n density on board

electric field

potential

Hint for HW #5.8

n AlGaAs | insulating GaAs

no electrons or holes
in depletion region

2DEG

undoped AlGaAs

$$\frac{dE}{dx} = \frac{q}{\varepsilon_p}\left(p - n + N_d^+ - N_a^-\right)$$

N_{d1}

$$\rightarrow \frac{dE}{dx} = \frac{q}{\varepsilon_p} n$$

charge density

$X_{dep,1}$

δ

$$\int_0^{Xdep2} \frac{dE}{dx} dx = \int_0^{Xdep2} \frac{q}{\varepsilon_p} n\, dx = \frac{q}{\varepsilon_p} n_s$$

$\varepsilon_{i,1}$

electric field

$$LHS = \int_{E(0)}^{E(Xdep2)} dE = E(Xdep2) - E(0) = E(0) \equiv \varepsilon_{i,2}$$

Part b, calculate $\varepsilon_{i,1}$

Then calculate $\varphi_{bi,1} \equiv \int_{-\infty}^{0} E(x)\,dx$

Then eliminate $X_{dep,1}$

$\varepsilon_{i,1} = \varepsilon_{i,2}$

$$\Rightarrow \varepsilon_{i,2} = \frac{q}{\varepsilon_p} n_s$$

Trianglular wells

$$E_1 \approx 4 \cdot 10^{-5} \left(\varepsilon_{i,2}\right)^{2/3} (V/cm)$$

V(x)

x

$$\left(E_f - E_1\right) = \frac{\hbar \pi}{m} n_s$$

$$\varepsilon_{i,2} = \frac{q}{\varepsilon_p} n_s$$

E_f

E_1

$$E_1 \approx 4 \cdot 10^{-5} \left(\frac{q}{\varepsilon_p} n_s\right)^{2/3} (V/cm)$$

Slope of this line is
$\varepsilon_{i,2}$
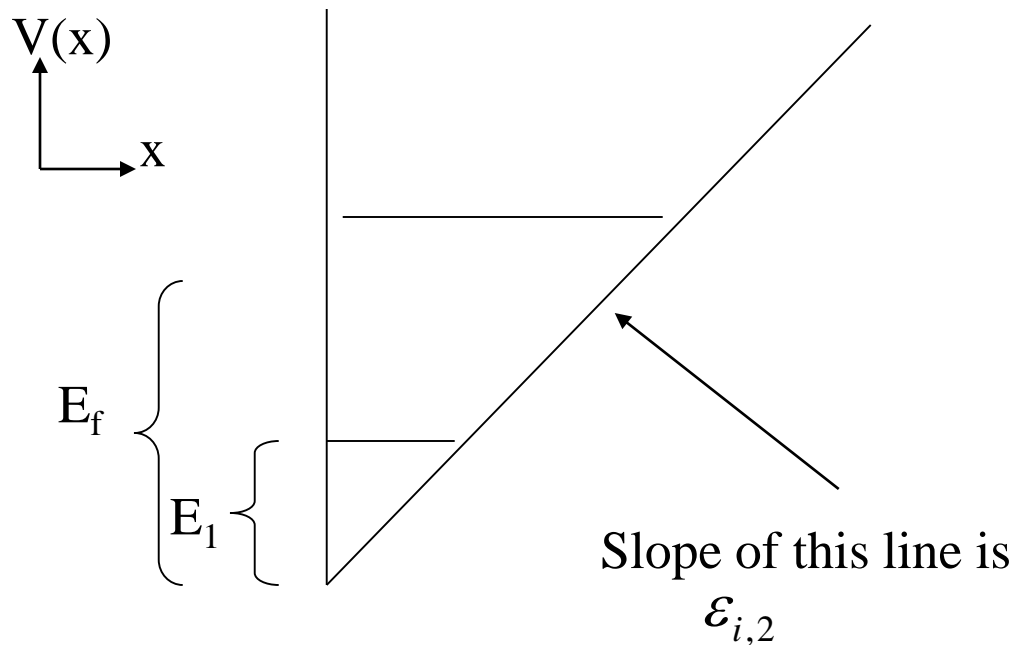
Discuss intuitively: adding electrons changes slope which changes Fermi energy.

That gives one relationship between E_f and n_s.
To solve for both of them, you need another relationship: HW 5.8

From HW 5.8

You will find:

$$n_s \propto \sqrt{\Delta E_c} \sqrt{N_d}$$

Want to engineer material so that ΔE_c large.
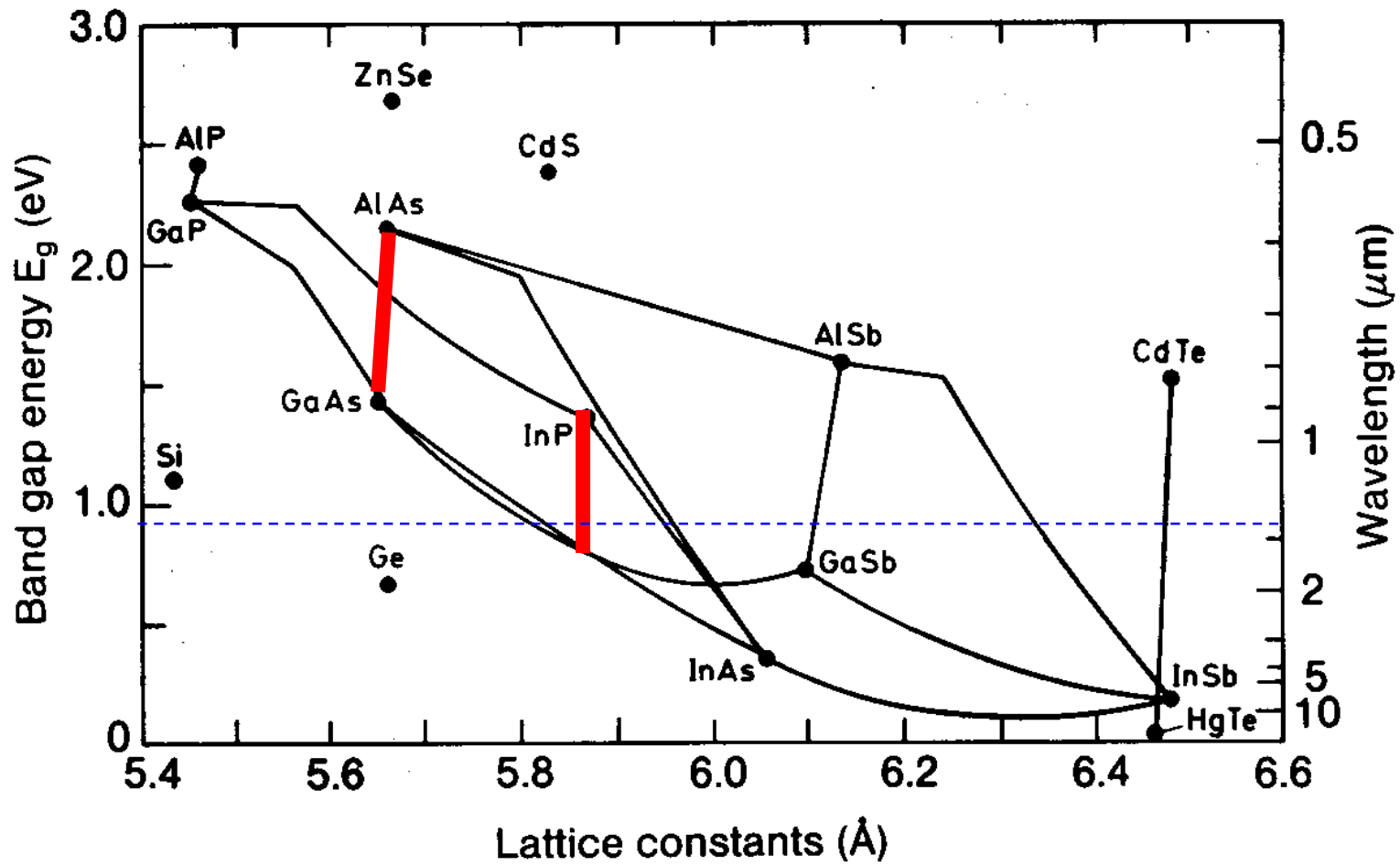For GaAs, there is a limit.

For $In_xGa_{1-x}As/In_xAl_{1-x}As$,
use strained layers to get larger ΔE_c (discuss).
(InP has higher mobility, peak velocity than GaAs.)

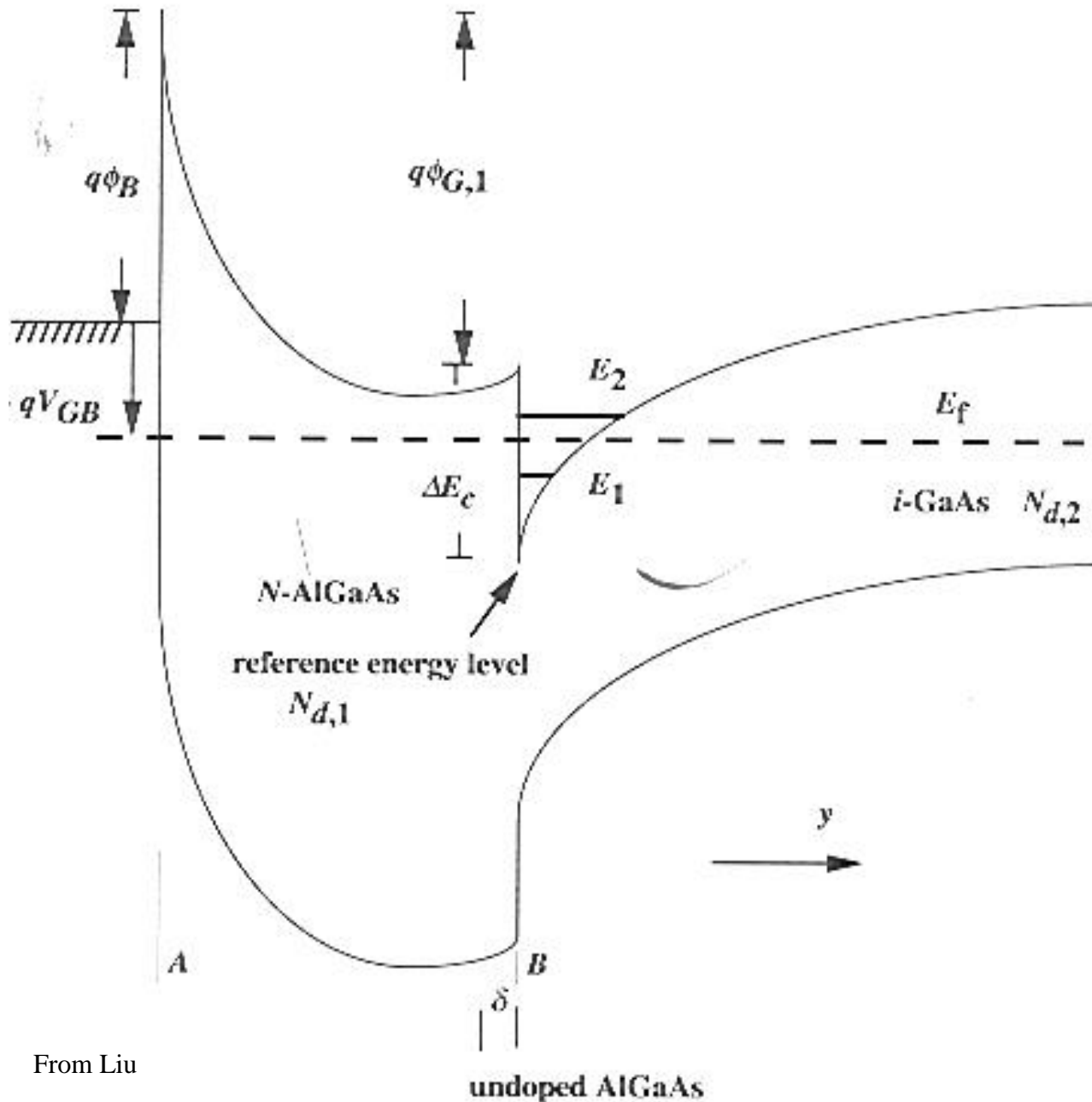Called pseudomorphic HEMT: pHEMT.

Band gaps

n_s vs E_f

After all that mumbo-jumbo, we know it is complicated.
We approximate it many times as:
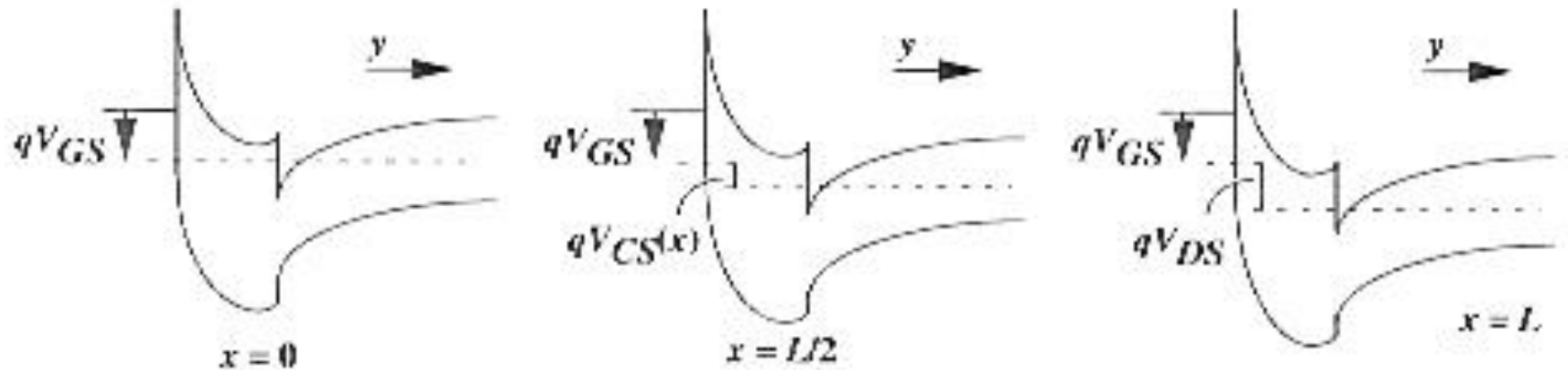
$$E_f(n_s) = E_{f,0} + a \cdot n_s$$

Gate

Gate changes E_f with respect to reference energy. This changes density.

From Liu

Vary gate voltage

Changes Fermi energy which changes density.

(Draw better pictures on board.)

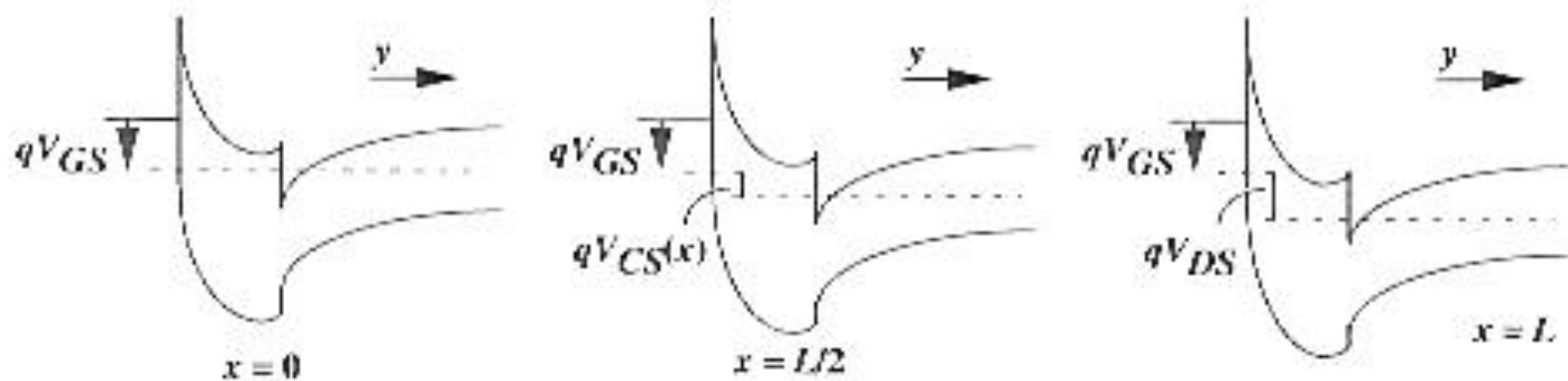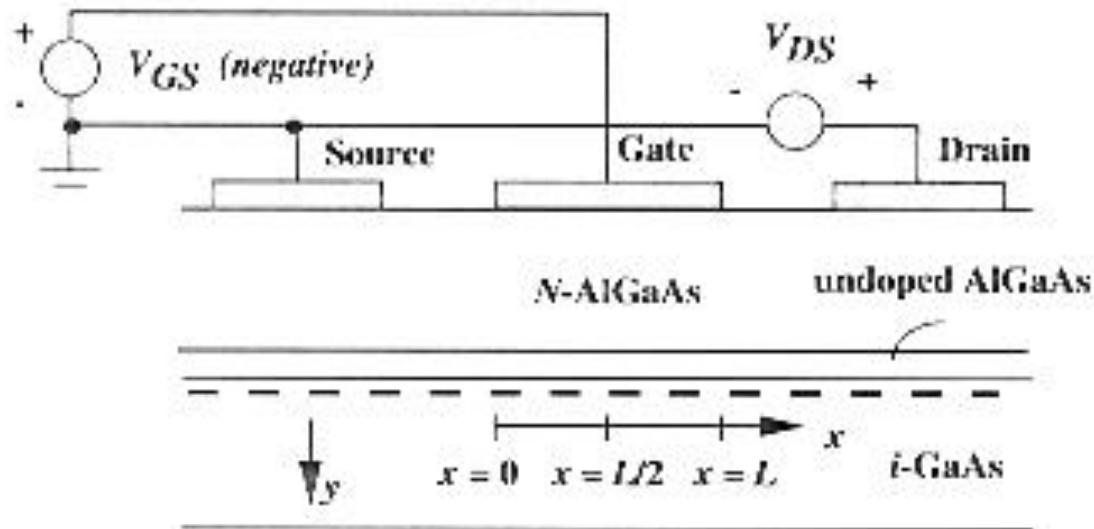From Liu.

Density

$$en_s = \frac{\varepsilon}{t_b + \varepsilon a / e^2}\left(V_{GB} - V_T\right)$$

$$V_T \equiv \phi_B + \frac{E_{f,0}}{e} - \frac{eN_{d,1}}{2\varepsilon}(t_b - \delta)^2 - \frac{\Delta E_c}{e}$$

HEMT analysis

From Liu.

Density

$$en_s = \frac{\varepsilon}{t_b + \varepsilon a / e^2}\left(V_{GB} - V_T\right)$$

$$\rightarrow en_s(x) = C_{ox}\left(V_{GS} - V_T - V_{CS}(x)\right)$$

$$C_{ox} \equiv \frac{\varepsilon}{t_b + \varepsilon a / e^2}$$

$$V_{CS}(0) = 0$$

$$V_{CS}(L) = V_{SD}$$

Current

J is 2d, n_s is 2d. (Discuss).

$$J = e \cdot \mu \cdot n_s \cdot E$$

$$I_D = J \cdot (width) = e \cdot \mu \cdot n(x) \cdot E(x) \cdot W$$

$$I_D = \mu \cdot C_{ox}\left(V_{GS} - V_T - V_{CS}(x)\right) \cdot E(x) \cdot W$$

$$= \mu \cdot C_{ox}\left(V_{GS} - V_T - V_{CS}(x)\right) \cdot \frac{\partial V_{CS}(x)}{\partial x} \cdot W$$
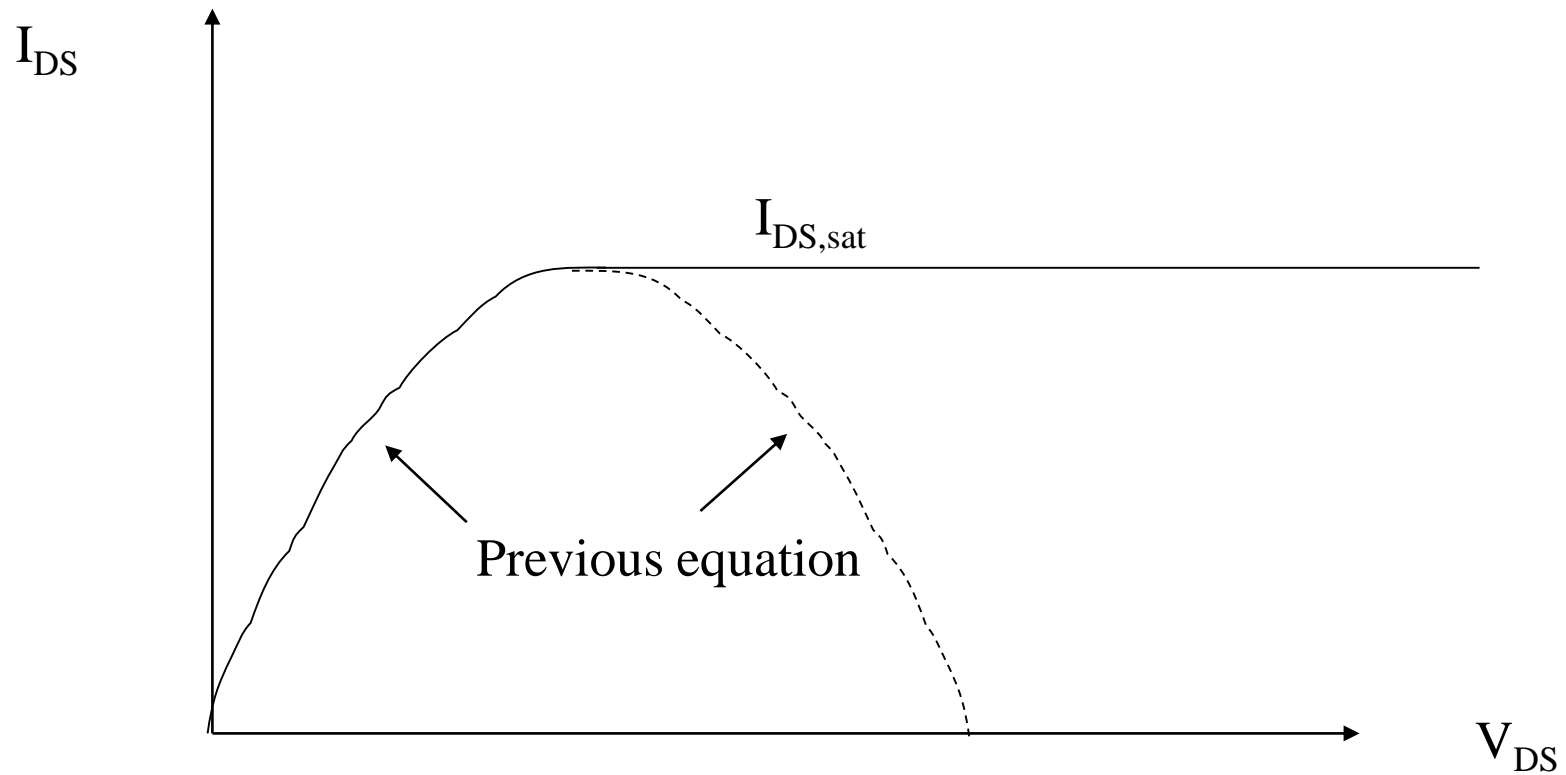
Integrating:

$$I_D = \mu \cdot C_{ox}\left(V_{GS} - V_T - V_{CS}(x)\right) \cdot \frac{\partial V_{CS}(x)}{\partial x} \cdot W$$

$$\int_0^L I_D dx = \int_0^L \mu \cdot C_{ox}\left(V_{GS} - V_T - V_{CS}(x)\right) \cdot \frac{\partial V_{CS}(x)}{\partial x} \cdot W dx$$

$$= \int_{V_{CS}(0)}^{V_{CS}(L)} \mu \cdot C_{ox}\left(V_{GS} - V_T - V_{CS}(x)\right) \cdot \partial V_{CS}(x) \cdot W = doable$$

$$I_D = \frac{W \cdot \mu \cdot C_{ox}}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

Conclusion:

I_{DS}

$I_{DS,sat}$

Previous equation

V_{DS}

Channel potential:

$$J = e \cdot \mu \cdot n_s \cdot E$$

$$I_D = J \cdot (width) = e \cdot \mu \cdot n(x) \cdot E(x) \cdot W$$

Since we are in 2d, no position dependent thickness b(x). Life is easier. It can be shown that:

$$V_{CS}(x) = \left(V_{GS} - V_T\right)\left[1 - \sqrt{1 - \frac{x}{L}\left(1 - \alpha^2\right)}\right]$$

$$\alpha \equiv \begin{cases} 1 - V_{DS}/V_{DS,sat} & \text{for} \quad V_{DS} < V_{DS,sat} \\ 0 & \text{for} \quad V_{DS} > V_{DS,sat} \end{cases}$$

Velocity saturation

- Just like MESFETs
- Important in short channel HEMTs
- Need to model channel as to regions: saturated and unsaturated
- Qualitative IVs are similar