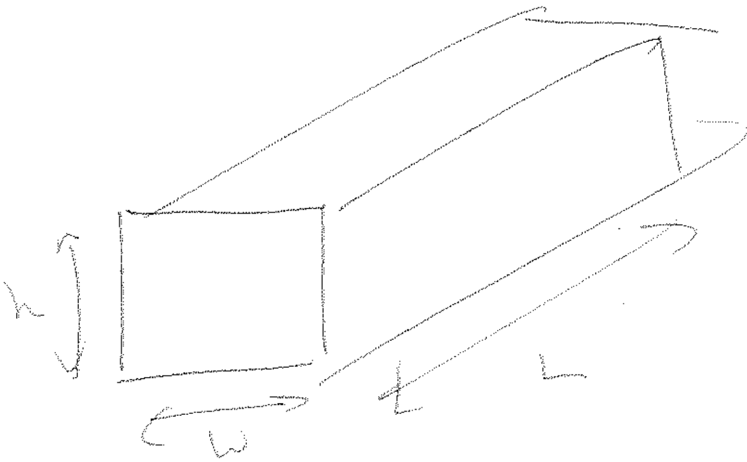
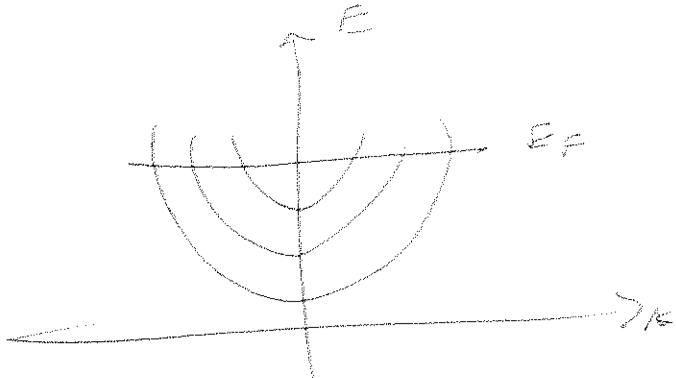
3d Sharvin



$$R = \frac{h}{2e^2}\,\frac{1}{M}$$

M = # of modes



$$E = \frac{\hbar^2}{2m}\left[\left(\frac{n_x \pi}{2L_x}\right)^2 + \left(\frac{n_y \pi}{2L_y}\right)^2 + \left(\frac{n_z \pi}{2L_z}\right)^2\right]$$

How to find M?

Need to find # of n_x, n_y combinations with $E(n_z=0, n_x, n_y) < E_F$.

This is "just" the 2d DOS:

In $k_x k_y$ space 2 states per $\left(\frac{2\pi}{L_x}\right)^2$

$$D(k)\,dk = \frac{2}{\left(2\pi/L_x\right)^2}\, 2\pi k\, dk$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}} \quad dk = \sqrt{\frac{2m}{\hbar^2}}\,\frac{dE}{2\sqrt{E}}$$

$$\Rightarrow D(E)\,dE = \frac{2}{\left(2\pi/L_x\right)^2}\, 2\pi \sqrt{\frac{2m\cancel{E}}{\hbar^2}}\sqrt{\frac{2m}{\hbar^2}}\,\frac{dE}{2\sqrt{\cancel{E}}}$$

$$= \frac{\cancel{2}}{\left(2\pi/L_x\right)^2}\, 2\pi\, \frac{2m}{\hbar^2}\,\frac{dE}{\cancel{2}}$$

$$= \frac{2^2 L_x^2}{\pi^2}\, 2\pi\, \frac{2m}{\hbar^2}\, dE$$

$$= \frac{m}{\pi\hbar^2}\, L_x^2\, dE$$

$$\#\text{ States (modes actually)} = \int_0^{E_F} \frac{m}{\pi\hbar^2} L_x^2\, dE = \frac{m}{\pi\hbar^2} L_x^2 E_F$$

$$M = \frac{m}{\pi \hbar^2} L_x^2 E_F$$

In 3d $\quad E_F = \frac{\hbar^2}{2m} (3\pi^2)^{2/3} n$ ← 3d density

Also $\quad E_F = \frac{\hbar^2}{2m} k_F^2 = \frac{\hbar^2}{2m} \left(\frac{2\pi}{\lambda_F}\right)^2$

$\Rightarrow$

$$M = \frac{\cancel{m}}{\cancel{\pi \hbar^2}} L_x^2 \frac{\cancel{\hbar^2}}{2\cancel{m}} \frac{(2\cancel{\pi})^2}{\lambda_F^2}$$

$$= 2 \frac{L_x^2}{\lambda_F^2}$$

$$\Rightarrow R_{\text{sharvin}} = \frac{h}{2e^2} \frac{\lambda_F^2}{L_x^2} \frac{1}{2}$$

Note: This HW is a "toy model" of graphene.
Does not get at 1) Valley degeneracy 2) 2D–1D nanoribbon effects

1) In graphene, we have a linear relationship between energy and momentum:

$$E = \hbar v_F k = \hbar v_F \sqrt{(k_x)^2 + (k_y)^2} = \hbar v_F \sqrt{\left(\frac{n_x \pi}{L_x}\right)^2 + \left(\frac{n_y \pi}{L_x}\right)^2} \quad \Rightarrow k = \frac{1}{v_F} E$$

↑ typo

$$\Rightarrow \frac{dk}{dE} = \frac{1}{v_F}$$

Derive the density of states vs. energy in graphene.

2) Now imagine you have a graphene nanoribbon. L_y is small. Calculate the density of states vs. energy of a 1d graphene nanoribbon.

$$E = \hbar v_F \sqrt{k_x^2 + k_{y0}^2} \qquad k_{y0} = \frac{\pi}{L_y}$$

1) $D(E)\,dE = D(k)\,dk$

$D(k)\,dk = \left[\frac{1 \text{ state}}{(\pi/L)^2} \times 2 \text{ (spin)}\right] \times$ area of disk of radius k in k-space $k_x > 0$ $k_y > 0$

$= \left[\frac{1}{(\pi/L)^2} \times \cancel{2}\right] \cancel{2}\pi k\,dk \frac{1}{\cancel{4}}$

$= L^2 \frac{1}{\pi} k\,dk = L^2 \frac{1}{\pi} \frac{E}{\hbar v_F} dk$

$\Rightarrow D(k) = L^2 \frac{1}{\pi} k = L^2 \frac{1}{\pi} E \frac{1}{\hbar v_F} \cancel{dk}$

$D(E) = D(k) \frac{dk}{dE} = \boxed{L^2 \frac{1}{\pi} E \frac{1}{\hbar v_F} \frac{1}{\hbar v_F} = D(E)}$

$$\boxed{\rho(E) = \frac{E}{\hbar^2 \pi v_F^2}}$$

2) $E = \hbar v_F k$

$D(E)\,dE = D(k)\,dk$

$D(k)\,dk = \left[\frac{1}{\pi/L} \times 2 \text{ (spin)}\right] \times$ distance in k-space between k, $k+dk$

$= \frac{2}{\pi/L} \times dk \Rightarrow D(k) = L \frac{2}{\pi}$

$D(E) = D(k) \frac{dk}{dE} = L \frac{2}{\pi} \frac{1}{v_F} \Rightarrow \boxed{\rho(E) = \frac{2}{\pi} \frac{1}{v_F}}$