EECS277C Winter 2017 **Midterm #1** Name: Solutions

2/2/2017 3:30 to 4:50 pm ID no.:

Professor Peter Burke

1	2	3	4	Total
/20	20	/20	/40	/100

Helpful constants for you:

$c = 3\ 10^{8}$ m/s
$e = 1.6\ 10^{-19}$ coulombs
$h = 6.63\ 10^{-34}$ J-s
$m = 9.1\ 10^{-31}$ kg
$k_B = 1.38\ 10^{-23}$ J/K
$h/e^2 = 25$ kΩ

2d density of states from HW:

$$\rho(E)dE = \frac{m}{\pi\hbar^2}dE$$

$$N(E) = \rho(E)\, L^2$$

1. [20 pts.] Find the relationship between the Fermi energy and the average energy of electrons in a 2d box.
2. [20 pts.] Same for the average wavelength, in a 2d box.

$$\langle E \rangle = \frac{1}{N}\int_0^\infty F(E)\, E\, N(E)\, dE = \frac{L^2}{N}\int_0^{E_F} E\, \rho(E)\, dE$$

$$= \frac{L^2}{N}\frac{m}{\pi\hbar^2}\frac{1}{2}E_F^2$$

Note E_F: $N = \int_0^{E_F} N(E)\, dE = \frac{m}{\pi\hbar^2}L^2 E_F \Leftrightarrow E_F = \frac{\pi\hbar^2}{m}\frac{N}{L^2}$

$$\Rightarrow \langle E \rangle = \frac{1}{2}E_F$$

$$E = \frac{\hbar^2}{2m}\left(\frac{2\pi}{\lambda}\right)^2 \Rightarrow \lambda = \sqrt{\frac{\hbar^2}{2mE}}\, 2\pi$$

$$\langle \lambda \rangle = \frac{1}{N}\int_0^\infty F(E)\, \lambda(E)\, N(E)\, dE$$

$$= \frac{L^2}{N}\int_0^{E_F} 2\pi\sqrt{\frac{\hbar^2}{2mE}}\frac{m}{\pi\hbar^2}dE = \frac{L}{N^2}2\pi\frac{m}{\pi\hbar^2}\sqrt{\frac{\hbar^2}{2m}}\int_0^{E_F}\frac{1}{\sqrt{E}}dE$$

$$= \frac{L}{N^2}2\pi\frac{m}{\pi\hbar^2}\sqrt{\frac{\hbar^2}{2m}}\, 2E_F^{1/2} = E_F^{-1}\, 2\pi\sqrt{\frac{\hbar^2}{2mE_F}}\, E_F\, 2 = 2\cdot 2\pi\sqrt{\frac{\hbar^2}{2mE_F}}$$

$$= 2\lambda_F$$

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3. [20 pts.] Calculate the Fermi energy for electrons in a hypothetical metal where each atom occupies 1x1x1 Ansgtrom.

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{L^3}\right)^{2/3}$$

$$\frac{N}{L^3} = \frac{1}{10^{-30} m^3}$$

$$E_F = \frac{\left(\frac{6.63\times10^{-34}\ J\text{-}s}{2\pi}\right)^2}{2\ 9.1\times10^{-31}\ kg}\left(2\pi^2\ 10^{30}\ m^{-3}\right)^{2/3}$$

$$= 36\ eV$$

4. [40 pts.] Derive the critical size a particle (box) has to have so that kT is larger than the energy level spacing of the lowest two energy levels.

$$\Delta E > kT$$

$$\Delta E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L^2}\right)^2\left[\left(2^2+1^2+1^2\right) - \left(1^2+1^2+1^2\right)\right]$$

$$= \frac{\hbar^2}{2m}\left(\frac{\pi}{L^2}\right)^2 3 > kT$$

$$L^2 < \frac{3\hbar^2}{2m}\frac{\pi^2}{kT} \qquad L < \sqrt{\frac{3h}{2m\,kT}}\ \pi\hbar$$

$$L < \sqrt{\frac{3}{2\ 9.1\times10^{-31}\ kg\ \ 1.38\times10^{-23}\ J/k\ \ 300k}}\ \pi\ \frac{6.63\times10^{-34}\ J\text{-}s}{2\pi}$$

$$\simeq 6\ nm$$