1. [20 pts.] Calculate the density of states in a 1 dimensional world.

1 dimension

$$\rho(E)dE = ?$$

We use:

$$\rho_k dk = \rho(E)dE$$

$$\rho_k dk = \frac{2}{\pi} dk$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow dk = \sqrt{\frac{2m}{\hbar^2}} \frac{dE}{2\sqrt{E}}$$

$$\boxed{\rho(E)dE = \frac{2}{\pi}\sqrt{\frac{2m}{\hbar^2}}\frac{dE}{2\sqrt{E}} = \frac{1}{\hbar\pi}\sqrt{\frac{2m}{E}}dE}$$

$$= \frac{\sqrt{m}}{\sqrt{E}} \frac{1}{\hbar} \frac{\sqrt{2}}{\pi}$$

[20 pts.] Consider a 1 nm x 1 nm x 1 nm metal nanoparticle. Find the energy level spacing between states at the Fermi energy. Hint: Calculate the DOS. Recall that the energy level spacing at the Fermi energy is not the same as the spacing between the lowest 2 energy levels.

$$N(E) \equiv \frac{\#\text{ states}}{\text{energy}} \Rightarrow \text{spacing between states} = \left.\frac{1}{N(E)}\right|_{E_F}$$

$$\rho = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} \equiv \frac{1}{L^3} N(E)$$

$$\Rightarrow \delta = \left.\frac{1}{N(E)}\right|_{E_F} = \left.\frac{1}{L^3\rho(E)}\right|_{E_F} = \frac{1}{L^3}\, 2\pi^2 \left(\frac{\hbar^2}{2m}\right)^{3/2} \frac{1}{\sqrt{E_F}}$$

Take $E_F = 1\text{eV} = 1.6\times10^{-19}\text{ J}$

$$\Rightarrow \delta = \frac{1}{(10^{-9}\text{m})^3}\, 2\pi^2 \left[\left[\frac{6.6\times10^{-34}}{2\pi}\right]^2 \frac{1}{2\cdot 9.1\cdot 10^{-31}\text{ Kg}}\right]^{3/2} \left[1.6\times10^{-19}\text{J}\right]^{-\frac{1}{2}}$$

$$= 10^{+9+9+9}\, 10^{-34-34-34}\, \underset{(46.5)}{10^{31\times\frac{3}{2}}}\, \underset{(9.5)}{10^{+19\times\frac{1}{2}}}\, 2^{1-3-\frac{3}{2}}\, \pi^{2-3}\, 9.1^{-3/2}\, 1.6^{-1/2}\ \text{J}$$

$$= 10^{-19}\, 2^{-3.5}\, \pi^{-1}\, 9.1^{-3/2}\, 1.6^{-1/2}\,\text{J} = 10^{-19}(0.088)(0.318)(0.036)(0.79)\text{J}$$

$$= 800\ 10^{-6}\, 10^{-19}\text{J} = 80\times10^{-24}\text{J} \simeq 0.5\text{ meV}$$