HW1 Problem 4; EECS 277C *Nanotechnology*

2 dimensions

$$N_k dk = ?$$

Volume of circular shell

$=2\pi kdk/4$

4 is for upper right quadrant

Number of states in area=
area x States/area

States/area = $1 / (\pi/L)^2$:

$$N_k dk = \left(2\pi kdk/4\right)\cdot\left(\frac{1}{(\pi/L)^2}\right)\cdot 2 = L^2\frac{kdk}{\pi}$$

$$\boxed{\rho_k dk \equiv \frac{N_k dk}{\text{area}} = \frac{kdk}{\pi}}$$

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2 dimensions

$$\rho(E)dE = ?$$

We use:

$$\rho_k dk = \rho(E)dE$$

$$\rho_k dk = \frac{kdk}{\pi}$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow dk = \sqrt{\frac{2m}{\hbar^2}} \frac{dE}{2\sqrt{E}}$$

$$\boxed{\rho(E)dE = \frac{m}{\pi\hbar^2} dE}$$

1) $E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 n_x^2 + n_y^2 + n_z^2$

$$\Delta E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2\left[\overset{3}{(1^2+1^2+1^2)} - \overset{6}{(2^2+1^2+1^2)}\right]$$

$$= \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 3$$

$h = 6.6\times10^{-34}$ J-s $\quad k = 1.38\times10^{-23}\frac{J}{K}$

$= 4.1\times10^{-15}$ eV-s $\quad m = 9.1\times10^{-31}$ kg

$\hbar = 10^{-34}$ J-s

$= 6.5\times10^{-16}$ eV-s

1m $\Delta E = \frac{(10^{-34}\text{ J-s})^2}{2\ 9.1 10^{-31}\text{ kg}}\left(\frac{\pi}{1m}\right)^2 3 =$

$= 1.6\times10^{-37}$ J $= 10^{-18}$ eV

1μm $\Delta E = 10^{-6}$ eV $=$ 1μeV

1nm $\Delta E = 10^{-0}$ eV $=$ 1eV

1Å $\Delta E =$ 10eV

$kT = 26$ meV @ $T = 300$K

EECS 277C Nanotechnology HW #2

1. Find the relationship between the Fermi energy and the average energy of electrons in a box.
2. Same for the average wavelength.
3. Find the Fermi wavelength of electrons in a typical metal, e.g, Cu.

1) $$\langle E \rangle = \frac{1}{N}\int_0^\infty F(E)\, E\, N(E)\, dE$$

$F(E)$ is appr. step function.

$$\cong \frac{1}{N}\int_0^{E_F} E\, N(E)\, dE$$

$$N(E) = \frac{L^3}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$$

$$\langle E \rangle = \frac{1}{N}\frac{L^3}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^{E_F} E^{3/2}\, dE$$

$$= \frac{1}{N}\frac{L^3}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\frac{2}{3}E_F^{5/2} \quad (**)$$

Recall $$E_F = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{L^3}\right)^{2/3} \quad (*)$$

Combining $(*)$, $(**)$ yields

$$\langle E \rangle = \frac{3}{5}E_F$$

2) $\langle\lambda\rangle = \frac{1}{N}\int_0^\infty \lambda\, F(E) N(E)\, dE$

$\cong \frac{1}{N}\int_0^{E_F} \lambda\, N(E)\, dE$ $\qquad E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{2\pi}{\lambda}\right)^2$

$= \frac{1}{N}\sqrt{\frac{\hbar^2}{2m}}\, 2\pi \int_0^{E_F} \frac{1}{\sqrt{E}} N(E)\, dE$ $\qquad \lambda = \sqrt{\frac{\hbar^2}{2mE}}\, 2\pi$

N(E) from before

$= \frac{1}{N}\sqrt{\frac{\hbar^2}{2m}}\, 2\pi\, \frac{L^3}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^{E_F} \frac{1}{\sqrt{E}}\sqrt{E}\, dE$

$= \frac{1}{N}\sqrt{\frac{\hbar^2}{2m}}\, \cancel{2\pi}\, \frac{L^3}{\cancel{2}\pi^{\cancel{2}}}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F$

$= \frac{L^3}{N}\frac{1}{\pi}\left(\frac{2m}{\hbar^2}\right) E_F$

$= \frac{\frac{1}{\cancel{\pi}}\frac{2m}{\hbar^2} E_F}{E_F^{3/2}\frac{1}{3\cancel{\pi}^{\cancel{2}}}\left(\frac{2m}{\hbar^2}\right)^{3/2}} = 3\pi E_F^{-1/2}\left(\frac{2m}{\hbar^2}\right)^{-1/2} = \langle\lambda\rangle$

If we define λ_F as $\sqrt{\frac{2mE_F}{\hbar^2}}$ we get

$\langle\lambda\rangle = \lambda_F \times \frac{3}{2}$

3) $$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{L^3}\right)^{2/3} \quad (*)$$

For copper, electron density = 1/atom
Using atom/m^3 for copper, we find

$$n \equiv \frac{N}{L^3} = 8.5\times 10^{28}\ \frac{\text{electrons}}{m^3}$$

Using $(*)$ $\Rightarrow$ $E_F \simeq 7\,eV$

HW1 Problem 6; EECS 277C *Nanotechnology*

Transmission prob:

$$\boxed{T = \left[1 + \frac{V_0^2 \sinh^2 [ka]}{4E(V_0 - E)} \right]^{-1}}$$

$$\boxed{k = \sqrt{2m(V_0 - E)/\hbar^2}}$$

$$\boxed{V_0 = 10eV}$$

$$\boxed{E = 5eV}$$

$$\boxed{T = 1.5\ 10^{-10}}$$

$$n\lambda = d\sin\theta$$

$$\Rightarrow \lambda = 1\,\mu m \ \sin 45^\circ \approx 0.7\,\mu m$$

$$d = 1\,\mu m \quad (10^3 \text{ lines/mm})$$

Not inch!

$$\lambda_{dB} \overset{10\text{ keV}\Rightarrow}{=} \sqrt{\frac{150}{10^4}}\,\mathring{A} = 0.12\,\mathring{A}$$

$$\sin\theta = \frac{\lambda}{d} = \frac{0.12\,\mathring{A}}{1\,\mu m} \cong 10^{-4}$$

$$\Rightarrow \theta \approx 10^{-4} \text{ radians}$$

$\sin\theta \approx \theta$ for small θ

$$\approx 0.006^\circ$$