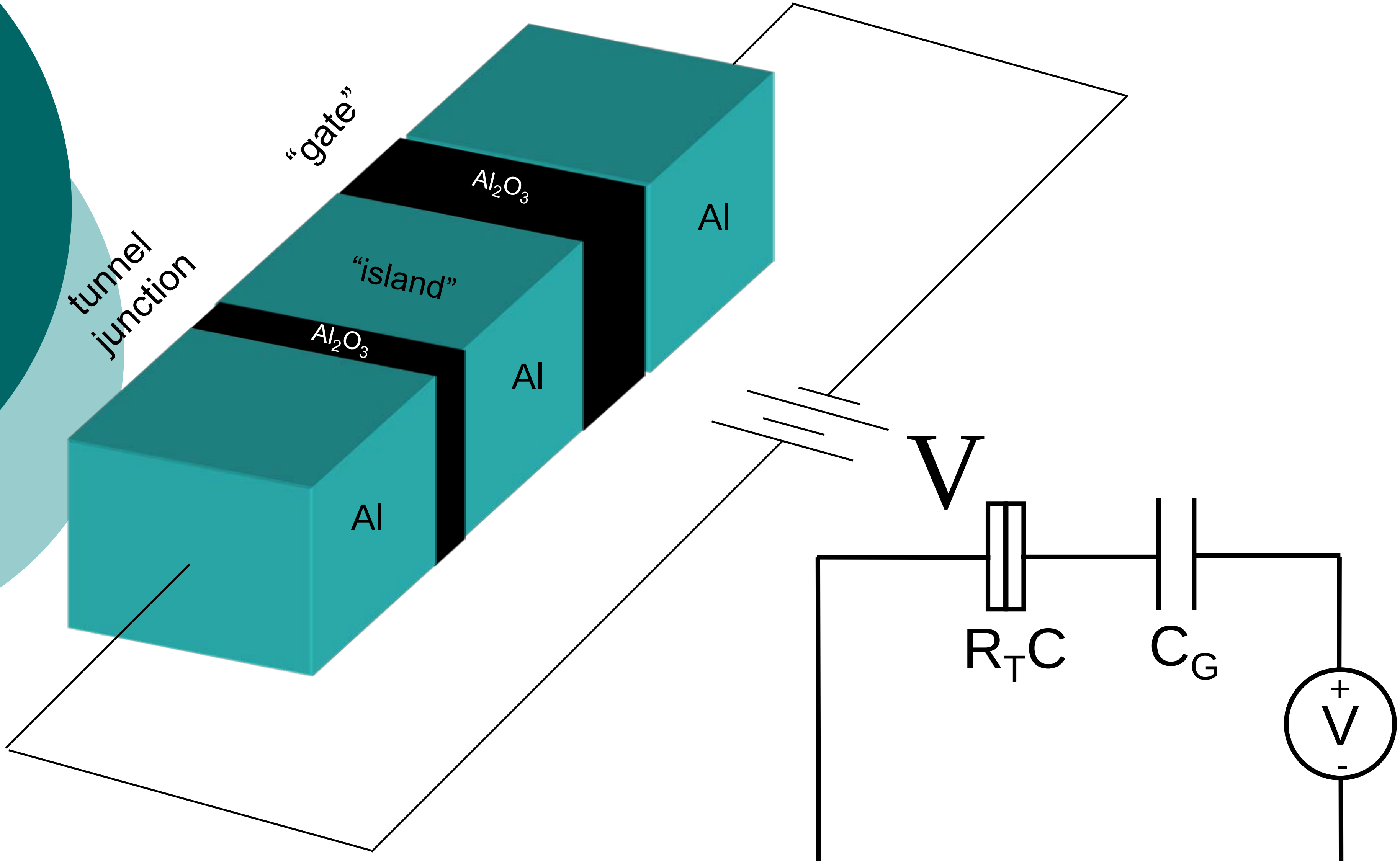
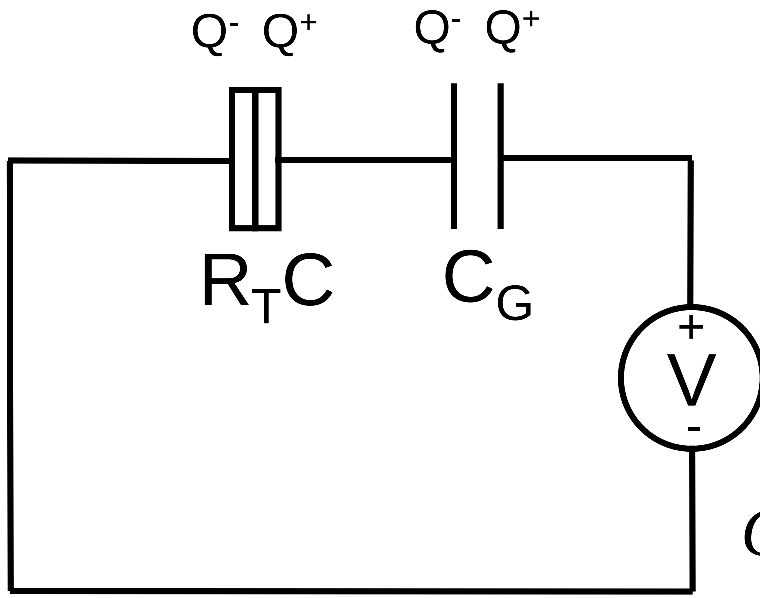
Lecture 6: Single electron box



Electrostatic energy (*no tunneling*)

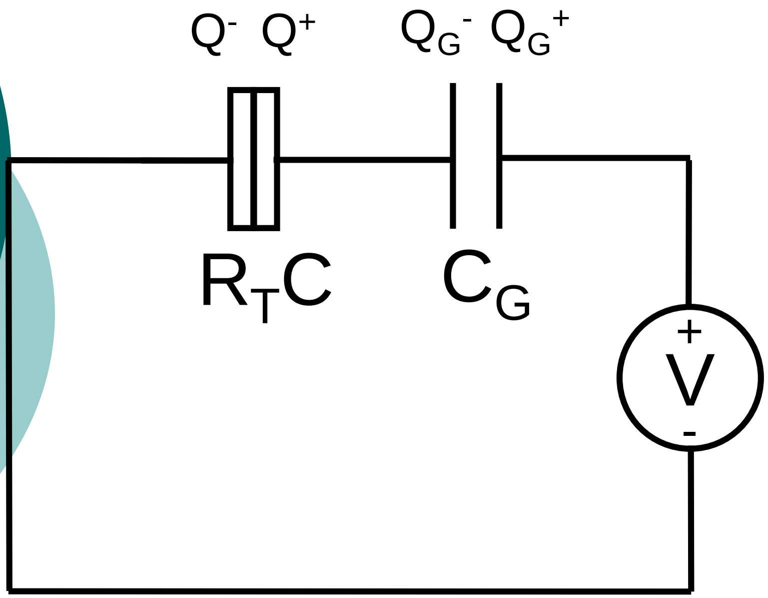


$$C_{series}{}^{-1} \equiv C^{-1} + C_G{}^{-1}$$

$$V = \frac{Q}{C} + \frac{Q}{C_G} = Q\left(\frac{1}{C} + \frac{1}{C_G}\right) = \frac{Q}{C_{series}}$$

$$E = \frac{Q^2}{2C} + \frac{Q^2}{2C_G} = \frac{Q^2}{2C_{series}} = \frac{1}{2}C_{series}V^2$$

Island charge



"Island charge":

$$Q_i = Q - Q_G$$

Kirchoff:

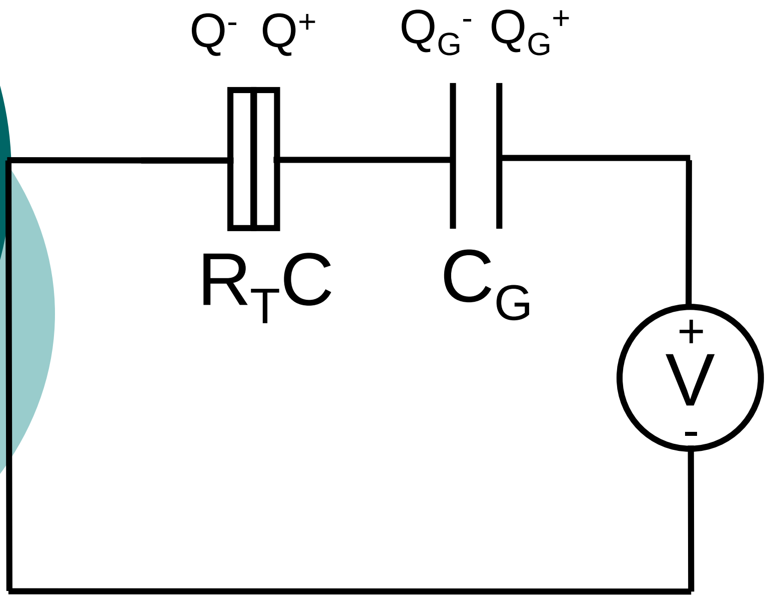
$$V = \frac{Q}{C} + \frac{Q_G}{C_G}$$

Solve for Q, Q_G:

$$Q = \frac{C(C_G V + Q_i)}{C + C_G}$$

$$Q_G = \frac{C_G(CV - Q_i)}{C + C_G}$$

Electrostatic energy (w/tunneling)



$$Q = \frac{C(C_G V + Q_i)}{C + C_G}$$

$$Q_G = \frac{C_G(CV - Q_i)}{C + C_G}$$

$$E = \frac{Q^2}{2C} + \frac{{Q_G}^2}{2C_G} = \frac{CC_G V^2 + {Q_i}^2}{2(C + C_G)}$$

Thermodynamics

Entropy:

$$S = S(E, V, N, \ldots)$$

Energy:

$$E = E(S, V, N, \ldots)$$

Entropy maximum ⇔ Energy minimum

Thermodynamic variables

Energy:

$$E = E(S, V, N, \ldots)$$

Temperature:

$$\frac{1}{T} \equiv \left.\frac{\partial E}{\partial S}\right|_{V,N,\ldots}$$

Pressure:

$$P \equiv -\left.\frac{\partial E}{\partial V}\right|_{E,N,\ldots}$$

Thermodynamic potentials

Energy:

$$E = E(S, V, N, \ldots)$$

Helmholtz potential (Helmholtz free energy):

$$F = E - TS$$

Minimized in presence of "reservoir" with temperature T.

Enthalpy:

$$H = E + PV$$

Minimized in presence of "reservoir" with pressure P.

Gibbs free energy:

$$G = E - TS + PV$$

Minimized in presence of "reservoir" with pressure P, temperature T.

Thermodynamic potentials for circuits

Energy:

$$E = E(S, V, N, Q, \ldots)$$

Gibbs free energy for electronic circuits:

$$G = E - Q_G V$$

Minimized in presence of
“reservoir” with voltage V.

electrostatic
energy

need to calculate

Q= how much charge has passed through the battery onto the gate
V= voltage of the battery

Free energy of single electron box:

$$G = E - Q_G V$$

From before:

$$E = \frac{CC_G V^2 + Q_i^2}{2(C + C_G)} \qquad Q_G = \frac{C_G(CV - Q_i)}{C + C_G}$$

$$G = \frac{CC_G V^2 + Q_i^2}{2(C + C_G)} - \frac{C_G(CV - Q_i)}{C + C_G} V$$

$$= \frac{1}{2}\frac{(C_G V + Q_i)^2}{C + C_G} - \frac{1}{2} C_G V^2$$

(Note: The last minus sign agrees with Lafarge thesis, but not Ferry textbook.)

Charge of island

From last slide:

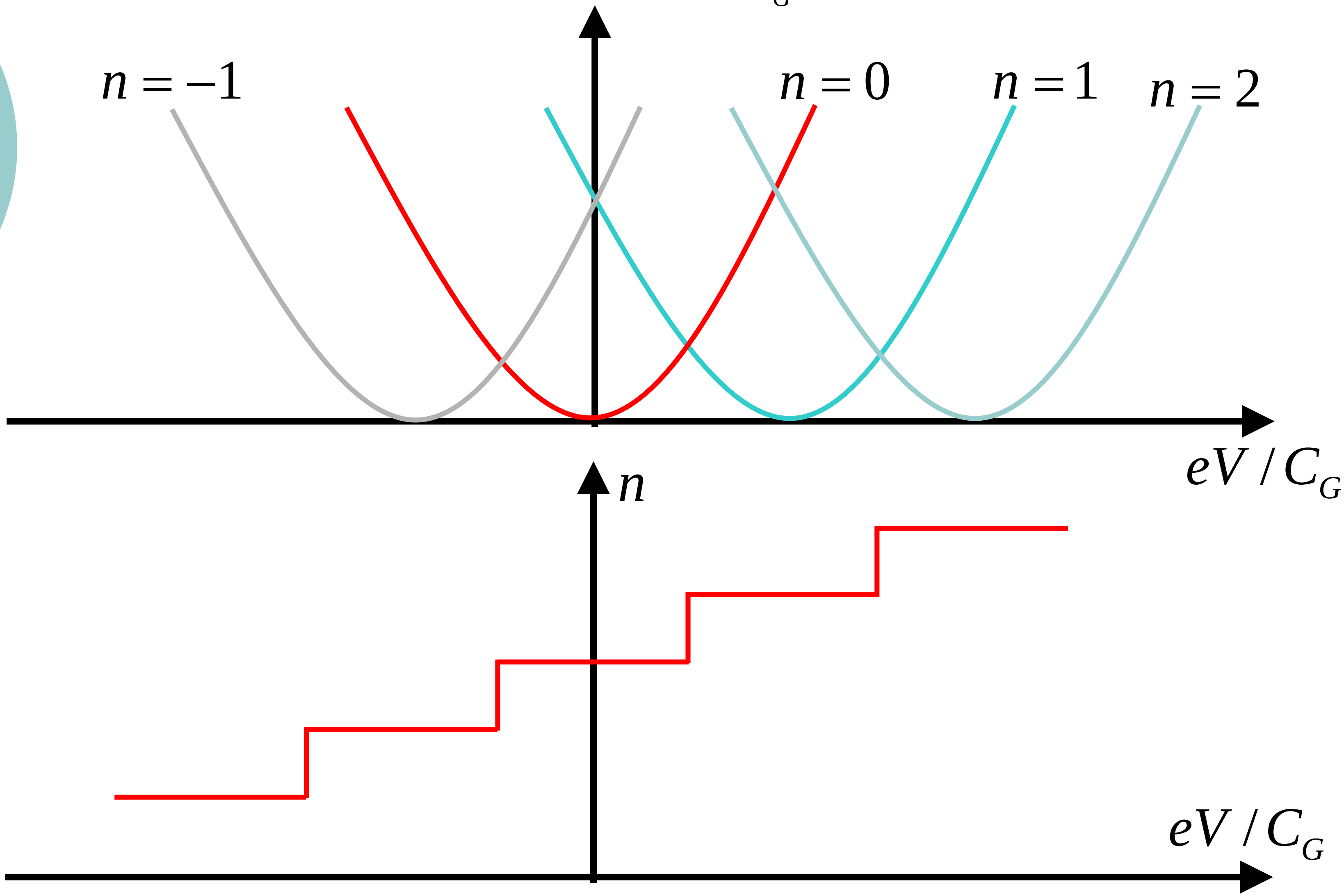
$$G = \frac{1}{2}\frac{(C_G V + Q_i)^2}{C + C_G} - \frac{1}{2} C_G V^2$$

$$Q_i = -ne$$ *only if* $R_T >> R_K$

$$G = \frac{1}{2}\frac{(C_G V - ne)^2}{C + C_G} + const$$

Charge of island

$$G = \frac{1}{2}\frac{(C_G V - ne)^2}{C + C_G}$$



Finite temperatures

Need
kT << $e^2/(C+C_G)$

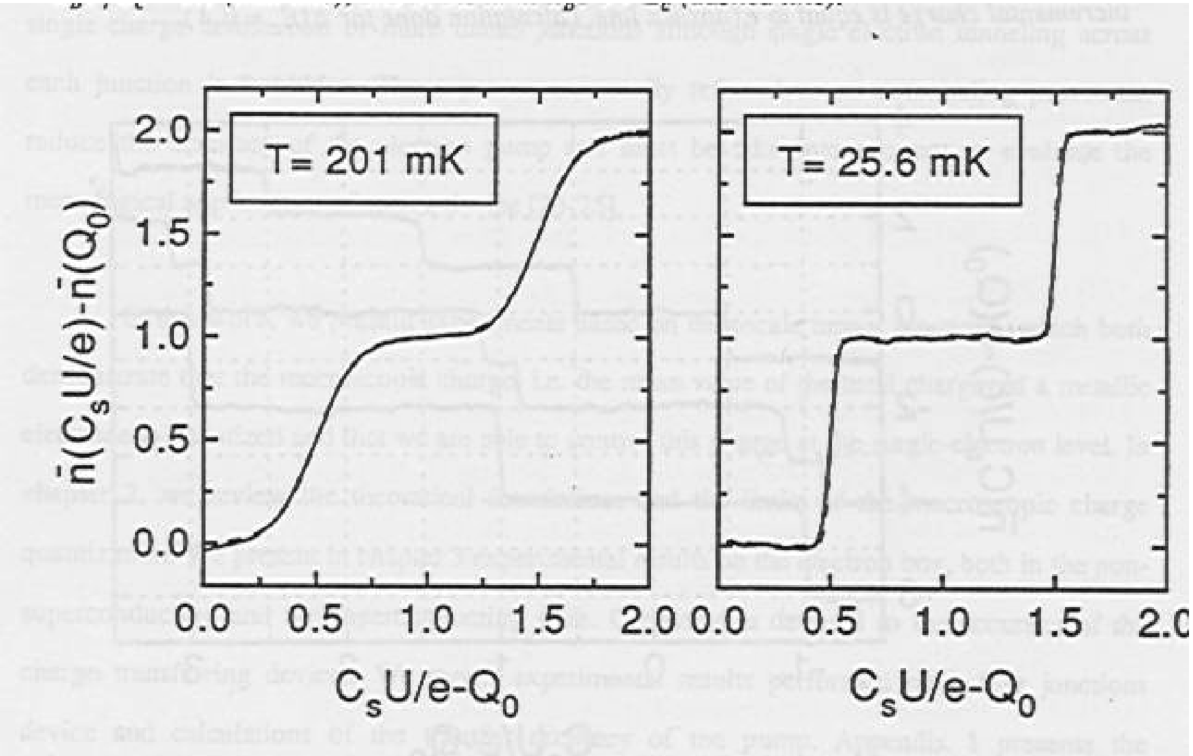


Fig. 1.5 b) *Solid lines: experimental variations of the average number $\bar{n}$ of excess electrons in in the island of an electron box. Dashed lines: theoretical calculations for an island capacitance $C_\Sigma = 0.8$ fF. The experimental parameters of the circuit are $C_s = 74$ aF and $C_c = 21$ aF. The quantity Q_0 denotes the random offset charge in the island.*

From Lafarge, PhD thesis, Universite Paris 6 (1993)

Quantum computing

- A single electron box has been proposed as a qu-bit
- |0> or |1> correspond to n or n+1 electrons
- Difficulty is fast (GHz) readout before decoherence sets in
- A superconducting box (for Cooper pairs) could have longer decoherence