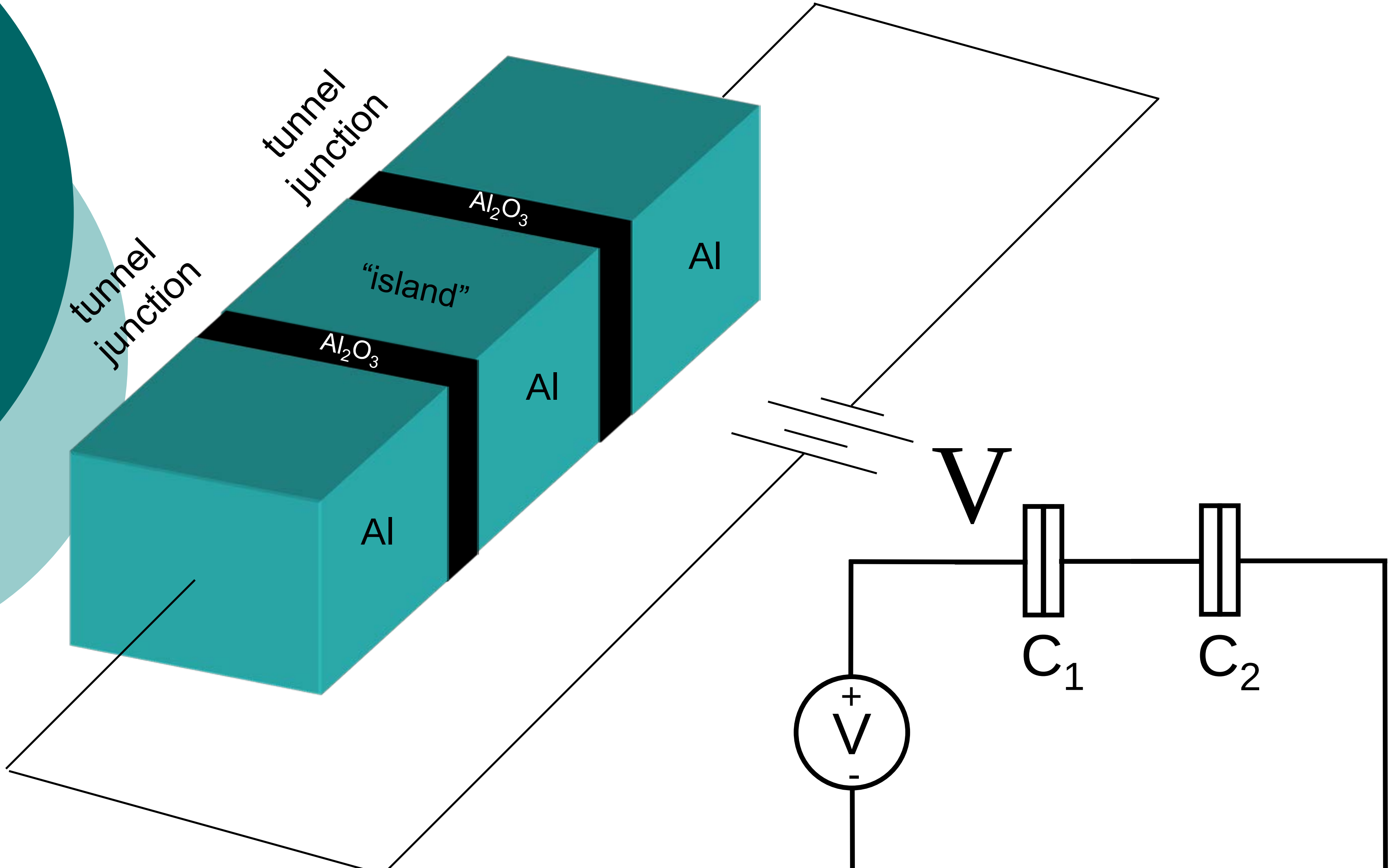
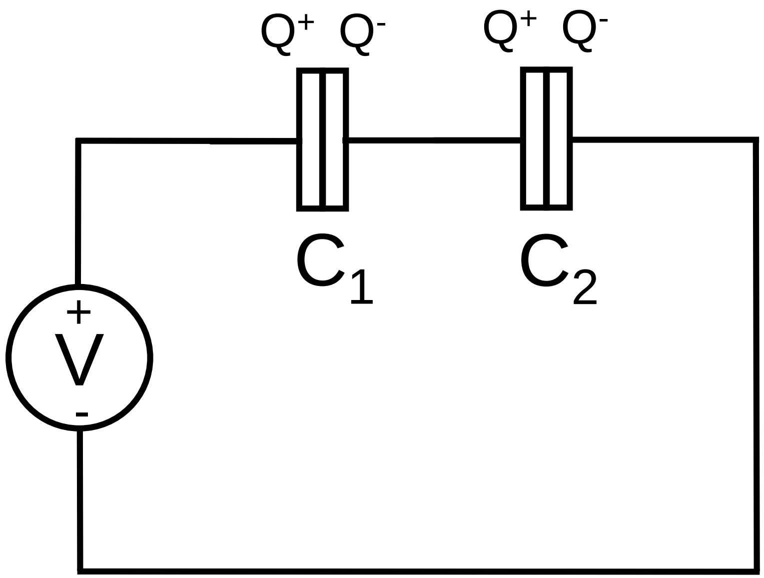
Lecture 7: Double tunnel junction



Electrostatic energy (*no tunneling*)

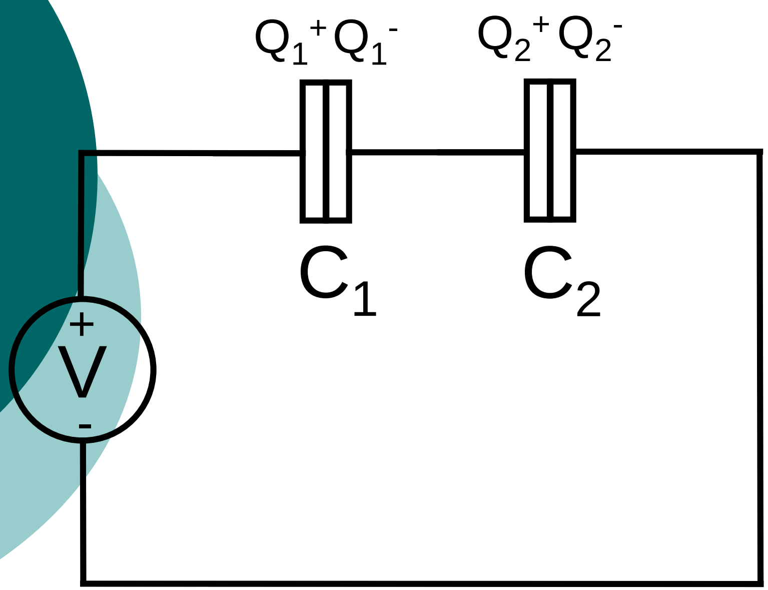


$$C_{series}{}^{-1} \equiv C^{-1} + C_G{}^{-1}$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q}{C_{series}}$$

$$E = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} = \frac{Q^2}{2C_{series}} = \frac{1}{2}C_{series}V^2$$

Island charge



"Island charge":

$$Q_i = Q_2 - Q_1$$

Kirchoff:

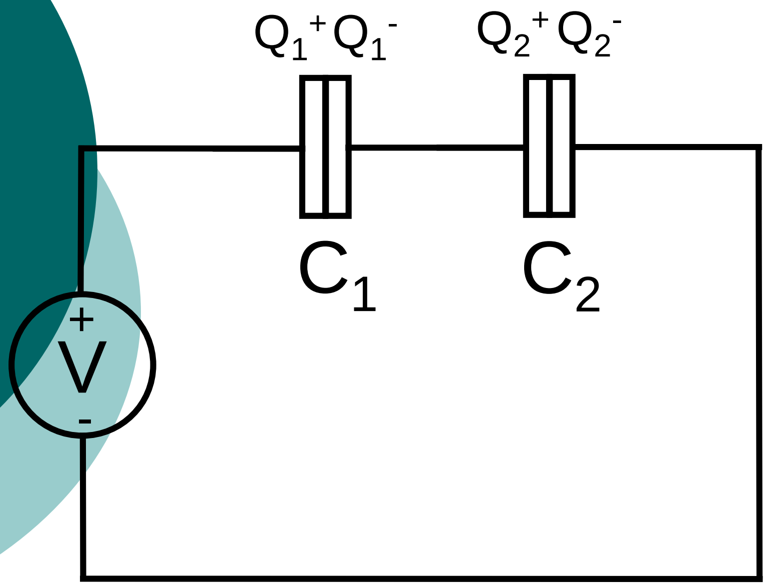
$$V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

Solve for Q, Q_G:

$$Q_1 = \frac{C_1(C_2V - Q_i)}{C_1 + C_2}$$

$$Q_2 = \frac{C_2(C_1V + Q_i)}{C_1 + C_2}$$

Electrostatic energy (*with tunneling*)



$$Q_1 = \frac{C_1(C_2 V - Q_i)}{C_1 + C_2}$$

$$Q_2 = \frac{C_2(C_1 V + Q_i)}{C_1 + C_2}$$

$$E = \frac{Q_1^{\ 2}}{2C_1} + \frac{Q_2^{\ 2}}{2C_2} = \frac{C_1 C_2 V^2 + Q_i^{\ 2}}{2(C_1 + C_2)}$$

Free energy :

$$G = E - Q_1 V$$

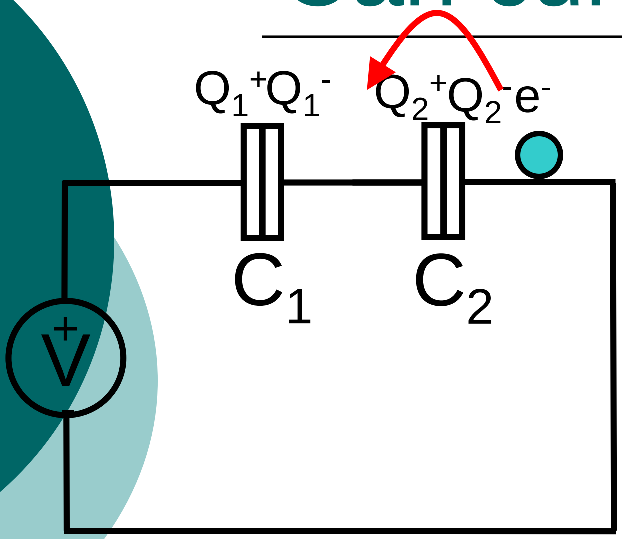
From before:

$$E = \frac{C_1 C_2 V^2 + Q_i^{\,2}}{2(C_1 + C_2)} \qquad Q_1 = \frac{C_1 (C_2 V - Q_i)}{C_1 + C_2}$$

$$G = \frac{C_1 C_2 V^2 + Q_i^{\,2}}{2(C_1 + C_2)} - \frac{C_1 (C_2 V - Q_i)}{C_1 + C_2} V$$

$$= \frac{1}{2} \frac{(C_1 V + Q_i)^2}{C_1 + C_2} - \frac{1}{2} C_1 V^2$$

Can current flow?



$$\Delta G = \Delta E - V \Delta Q_1$$

Before:

$$Q_i = -n_0 e$$

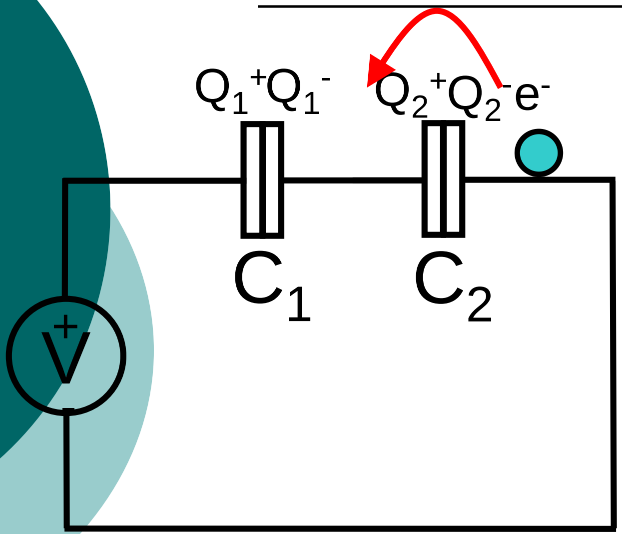
After:

$$Q_i = -n_0 e - e$$

$$\Delta E = \frac{C_1 C_2 V^2 + (-n_0 e)^2}{2(C_1 + C_2)} - \frac{C_1 C_2 V^2 + (-n_0 e - e)^2}{2(C_1 + C_2)} =$$

$$= \frac{(-n_0 e)^2}{2(C_1 + C_2)} - \frac{(-n_0 e - e)^2}{2(C_1 + C_2)} = \frac{-2 n_0 e^2 - e^2}{2(C_1 + C_2)}$$

Can current flow?



$$\Delta G = \Delta E - V \Delta Q_1$$

Before:

$$Q_i = -n_0 e$$

$$Q_1 = \frac{C_1(C_2 V - (-n_0 e))}{C_1 + C_2}$$

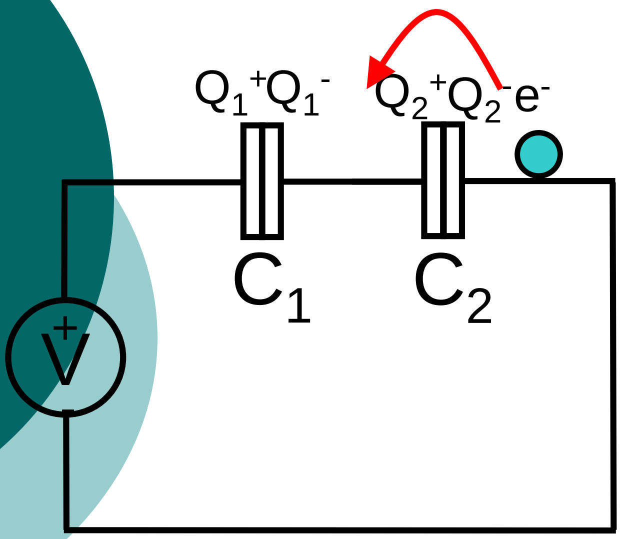
After:

$$Q_i = -n_0 e - e$$

$$Q_1 = \frac{C_1(C_2 V - (-n_0 e - e))}{C_1 + C_2}$$

$$\Delta Q_1 = -\frac{C_1 e}{C_1 + C_2}$$

Can current flow?



$$\Delta G = \Delta E - V \Delta Q_1$$

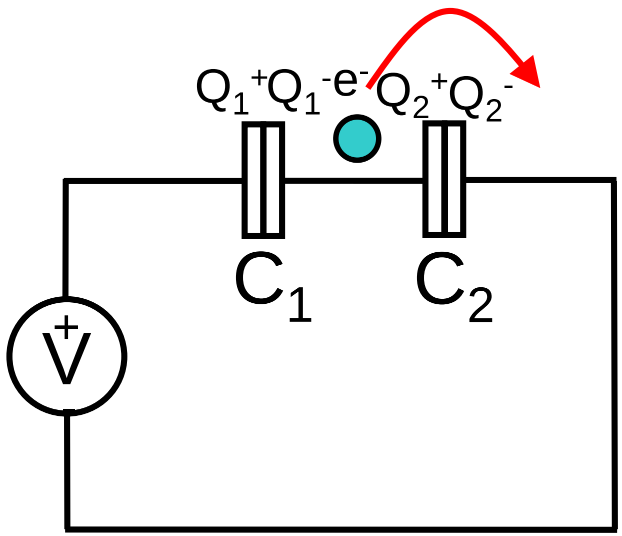
$$\Delta E = \frac{-2n_0 e^2 - e^2}{2(C_1 + C_2)}$$

$$\Delta Q_1 = -\frac{C_1 e}{C_1 + C_2}$$

$$\Delta G = \frac{-2n_0 e^2 - e^2}{2(C_1 + C_2)} + V\frac{C_1 e}{C_1 + C_2} = \frac{e}{C_1 + C_2}\left[-n_0 e - \frac{e}{2} + C_1 V\right] > 0$$

$$V > \frac{e}{C_1}\left(n_0 + \frac{1}{2}\right)$$

Similarly:

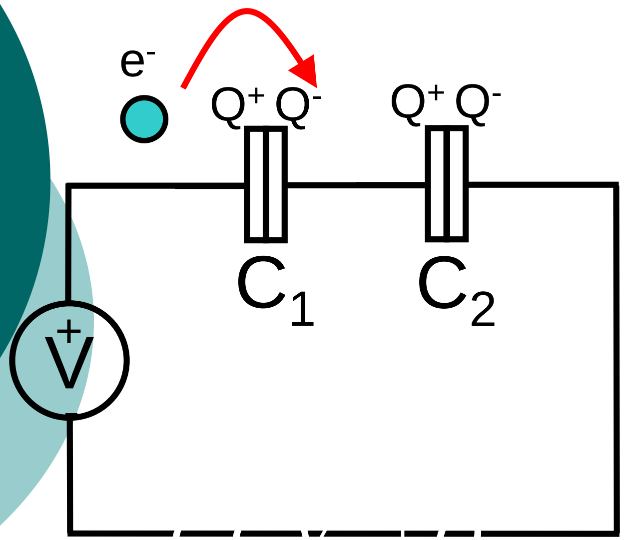


Allowed only if:

$$V < \frac{e}{C_1}\left(n_0 - \frac{1}{2}\right)$$

n_0 is the number of electrons on the island *before* the tunnel event.

Can current flow?



$$\Delta G = \Delta E - V \Delta Q_1$$

Before:

$$Q_i = -n_0 e$$

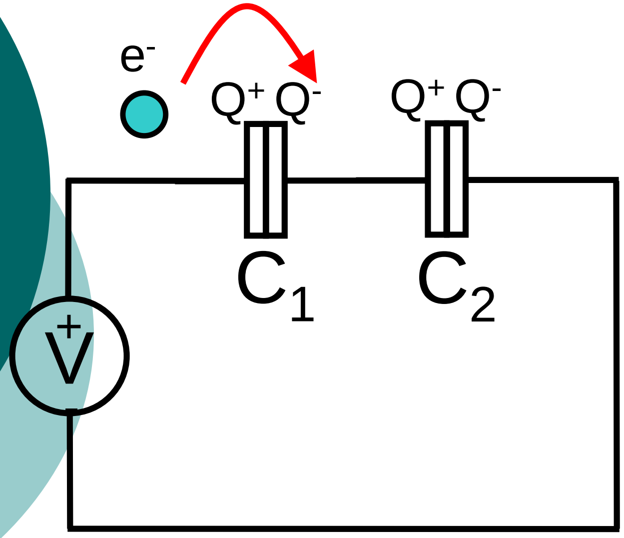
After:

$$Q_i = -n_0 e - e$$

$$\Delta E = \frac{C_1 C_2 V^2 + (-n_0 e)^2}{2(C_1 + C_2)} - \frac{C_1 C_2 V^2 + (-n_0 e - e)^2}{2(C_1 + C_2)} =$$

$$= \frac{(-n_0 e)^2}{2(C_1 + C_2)} - \frac{(-n_0 e - e)^2}{2(C_1 + C_2)} = \frac{-2 n_0 e^2 - e^2}{2(C_1 + C_2)}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1$$

Before:

$$Q_i = -n_0 e \qquad Q_1 = \frac{C_1(C_2V - (-n_0 e))}{C_1 + C_2}$$

After:

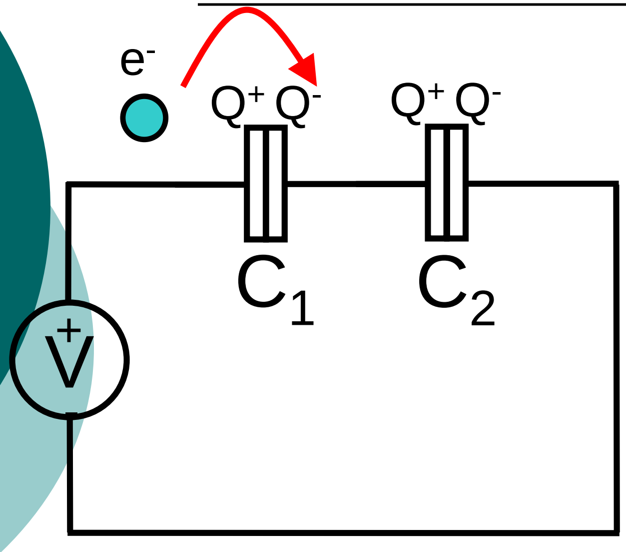
$$Q_i = -n_0 e - e \qquad Q_1 = \frac{C_1(C_2V - (-n_0 e - e))}{C_1 + C_2}$$

$$\Delta Q_{1,polarization} = -\frac{C_1 e}{C_1 + C_2}$$

"But…" $\Delta Q_{1,tunnel} = e$

$$\Delta Q_{1,total} = e - \frac{C_1 e}{C_1 + C_2} = \frac{C_1 + C_2}{C_1 + C_2} e - \frac{C_1 e}{C_1 + C_2} = \frac{C_2 e}{C_1 + C_2}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1$$

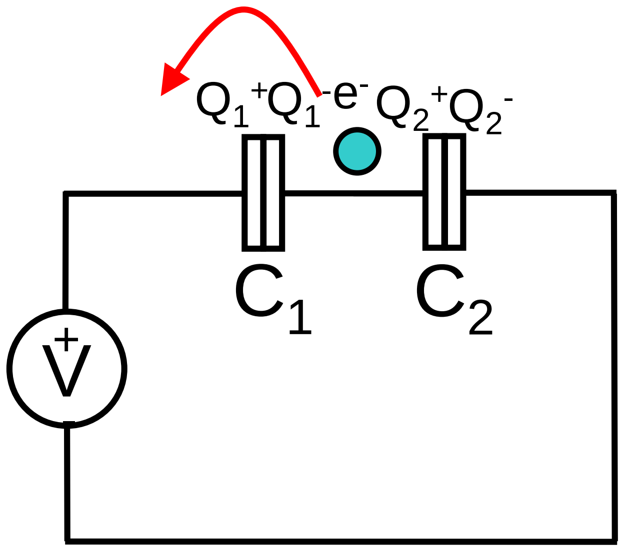
$$\Delta Q_{1,total} = \frac{C_2 e}{C_1 + C_2}$$

$$\Delta E = \frac{-2n_0 e^2 - e^2}{2(C_1 + C_2)}$$

$$\Delta G = \frac{-2n_0 e^2 - e^2}{2(C_1 + C_2)} + V\frac{C_2 e}{C_1 + C_2} = \frac{e}{C_1 + C_2}\left[-n_0 e - \frac{e}{2} - C_2 V\right] > 0$$

$$V < -\frac{e}{C_2}\left(n_0 + \frac{1}{2}\right)$$

Similarly:

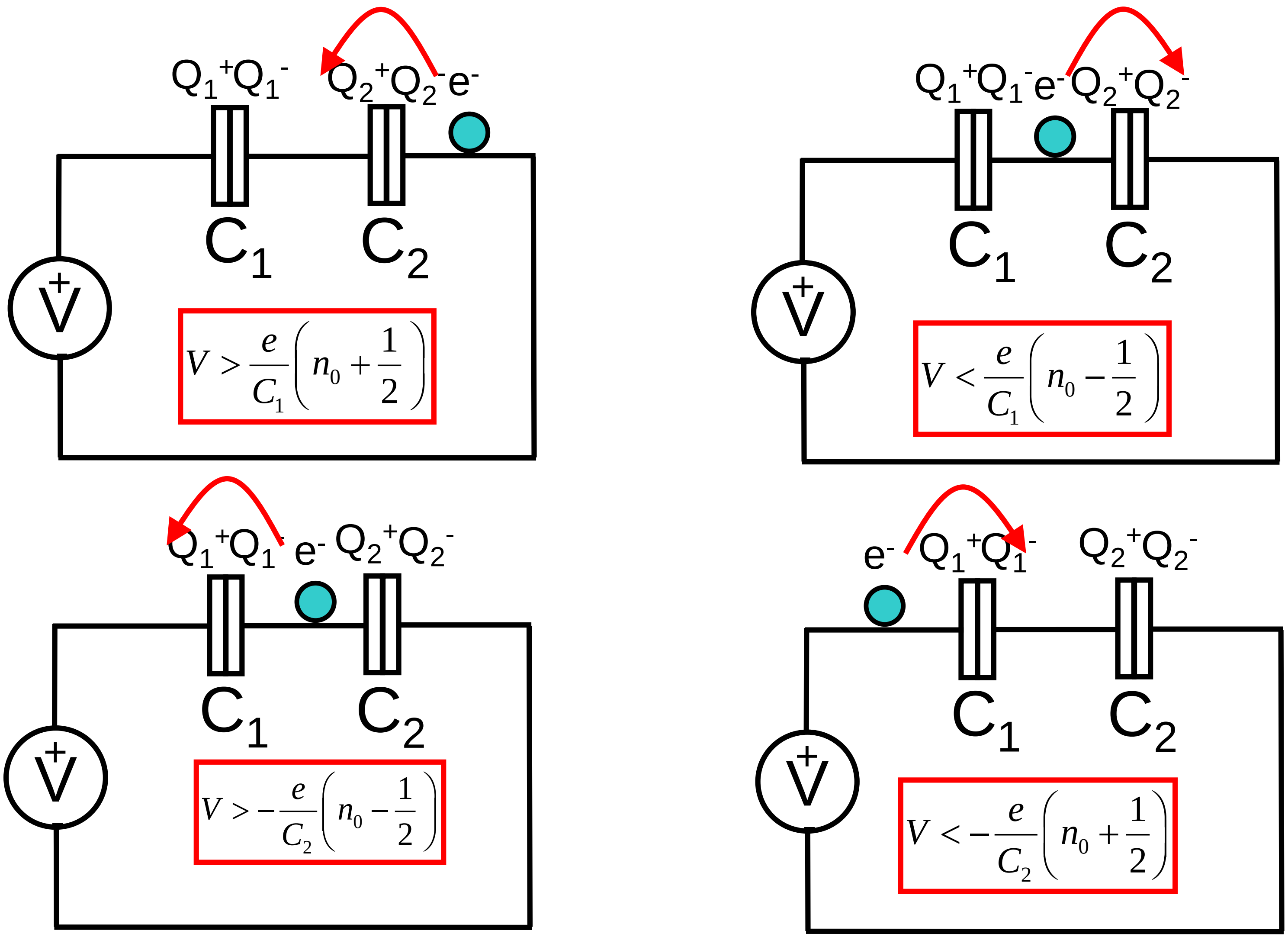


Allowed only if:

$$V > -\frac{e}{C_2}\left(n_0 - \frac{1}{2}\right)$$

n_0 is the number of electrons on the island *before* the tunnel event.

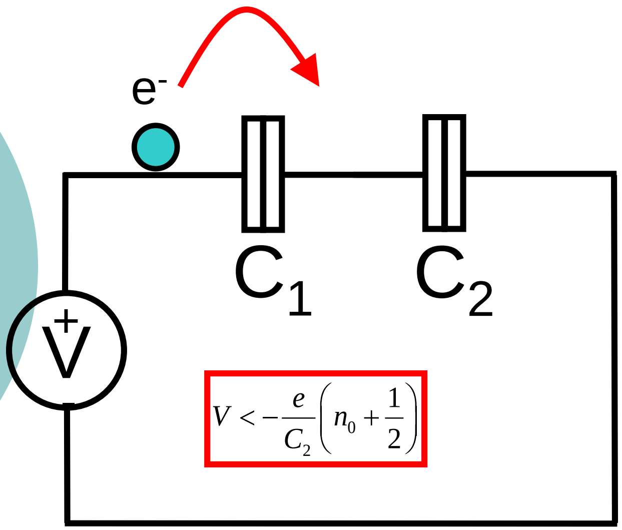
Summary



$Q_1^+ Q_1^-$ $Q_2^+ Q_2^-$ e^-
C_1 C_2
V
$V > \frac{e}{C_1}\left(n_0 + \frac{1}{2}\right)$
$Q_1^+ Q_1^-$ e^- $Q_2^+ Q_2^-$
C_1 C_2
V
$V < \frac{e}{C_1}\left(n_0 - \frac{1}{2}\right)$
$Q_1^+ Q_1^-$ e^- $Q_2^+ Q_2^-$
C_1 C_2
V
$V > -\frac{e}{C_2}\left(n_0 - \frac{1}{2}\right)$
e^- $Q_1^+ Q_1^-$ $Q_2^+ Q_2^-$
C_1 C_2
V
$V < -\frac{e}{C_2}\left(n_0 + \frac{1}{2}\right)$

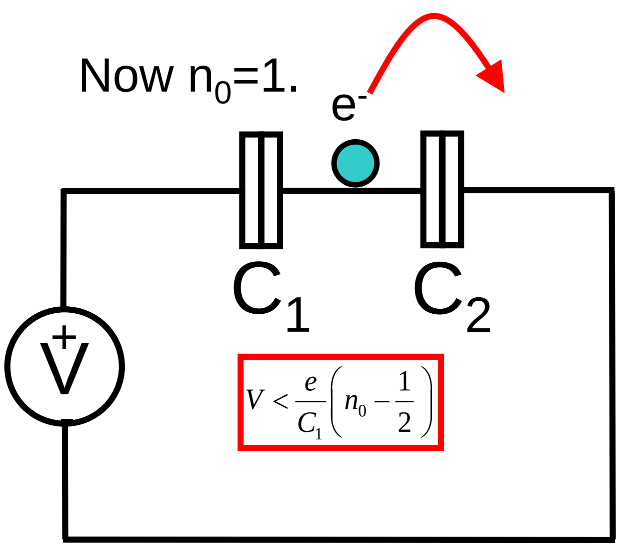
Current

Let n_0=0. Let C_1= C_2



$$V < -\frac{e}{2C_2}$$

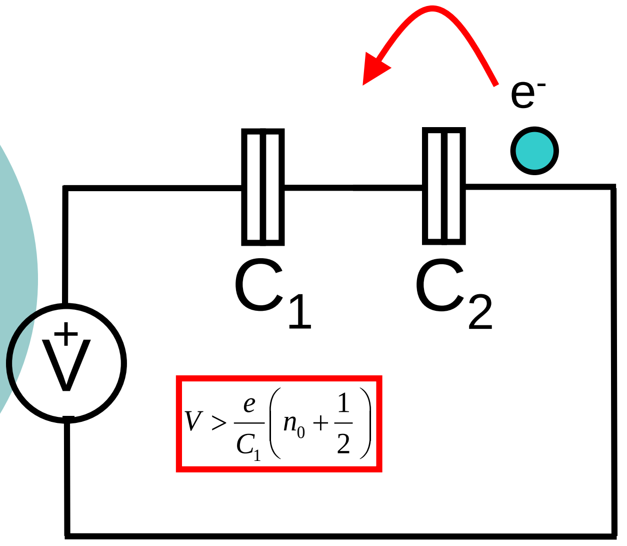
Now n_0=1.



$$V < \frac{e}{2C_2}$$

Current

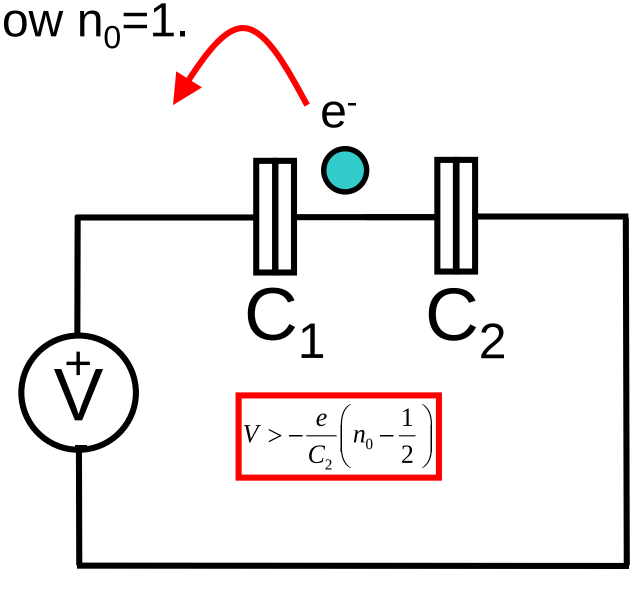
Let n_0=0. Let C_1= C_2



$$V > \frac{e}{C_1}\left(n_0 + \frac{1}{2}\right)$$

$$V > \frac{e}{2C_2}$$

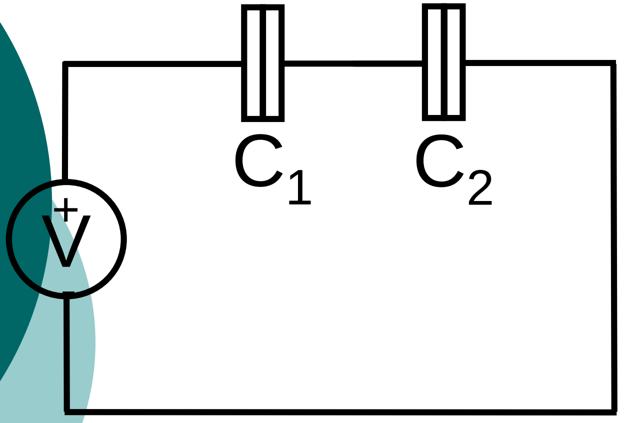
Now n_0=1.



$$V > -\frac{e}{C_2}\left(n_0 - \frac{1}{2}\right)$$

$$V > -\frac{e}{2C_2}$$

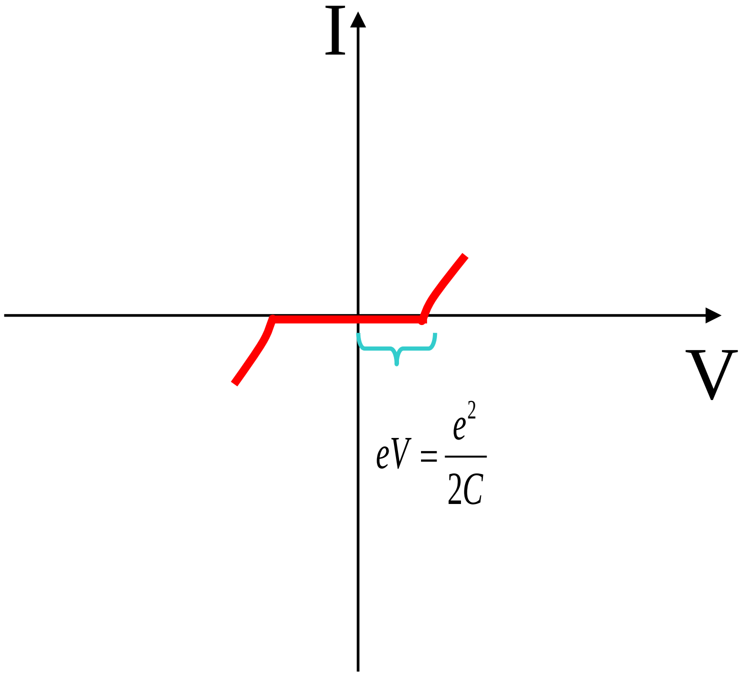
Coulomb blockade



Let $C_1 = C_2$

No current:

$$-\frac{e}{2C_2} < V < \frac{e}{2C_2}$$

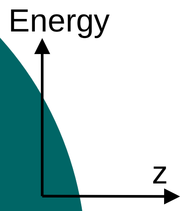


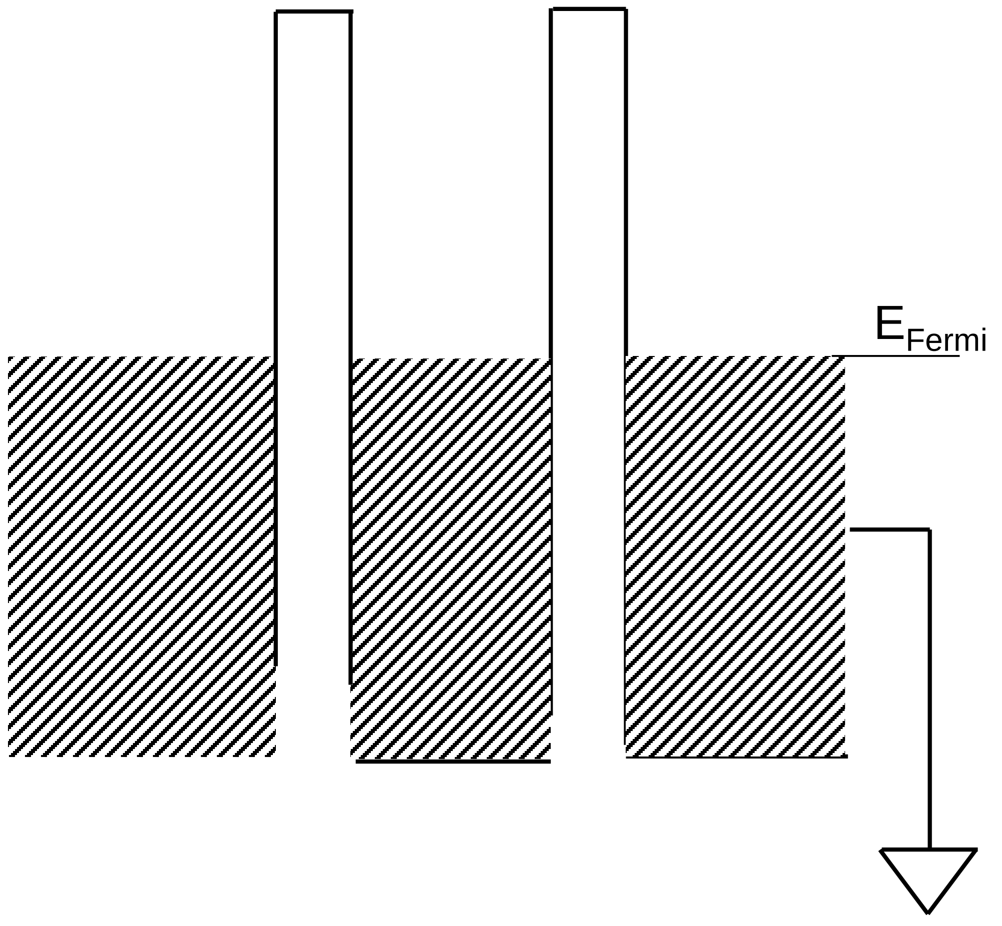
From quantum circuit theory, works even when voltage biased.

In single junction, Coulomb blockade *hard* to observe.

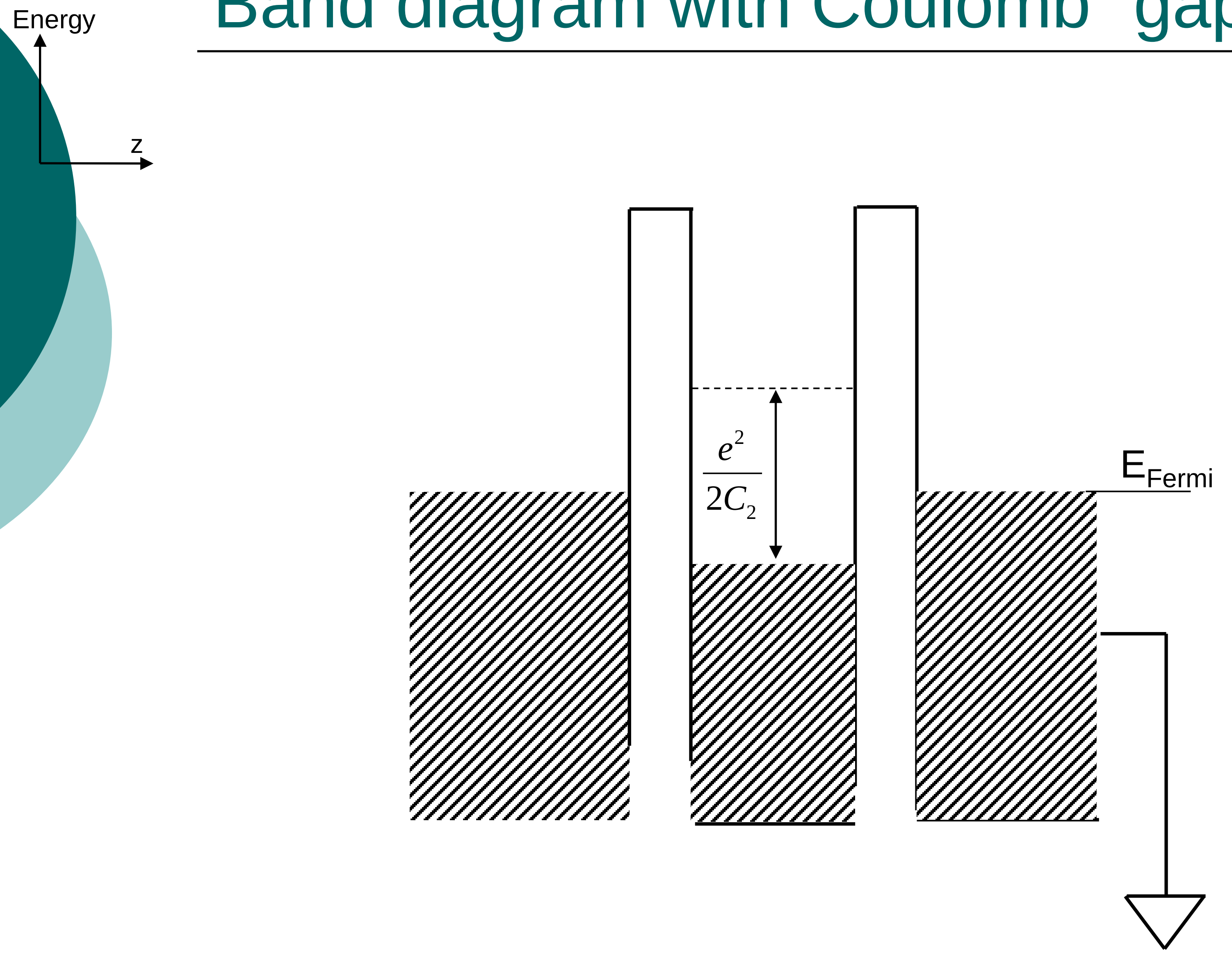
In double junction, Coulomb blockade *easy* to observe.

Band diagram

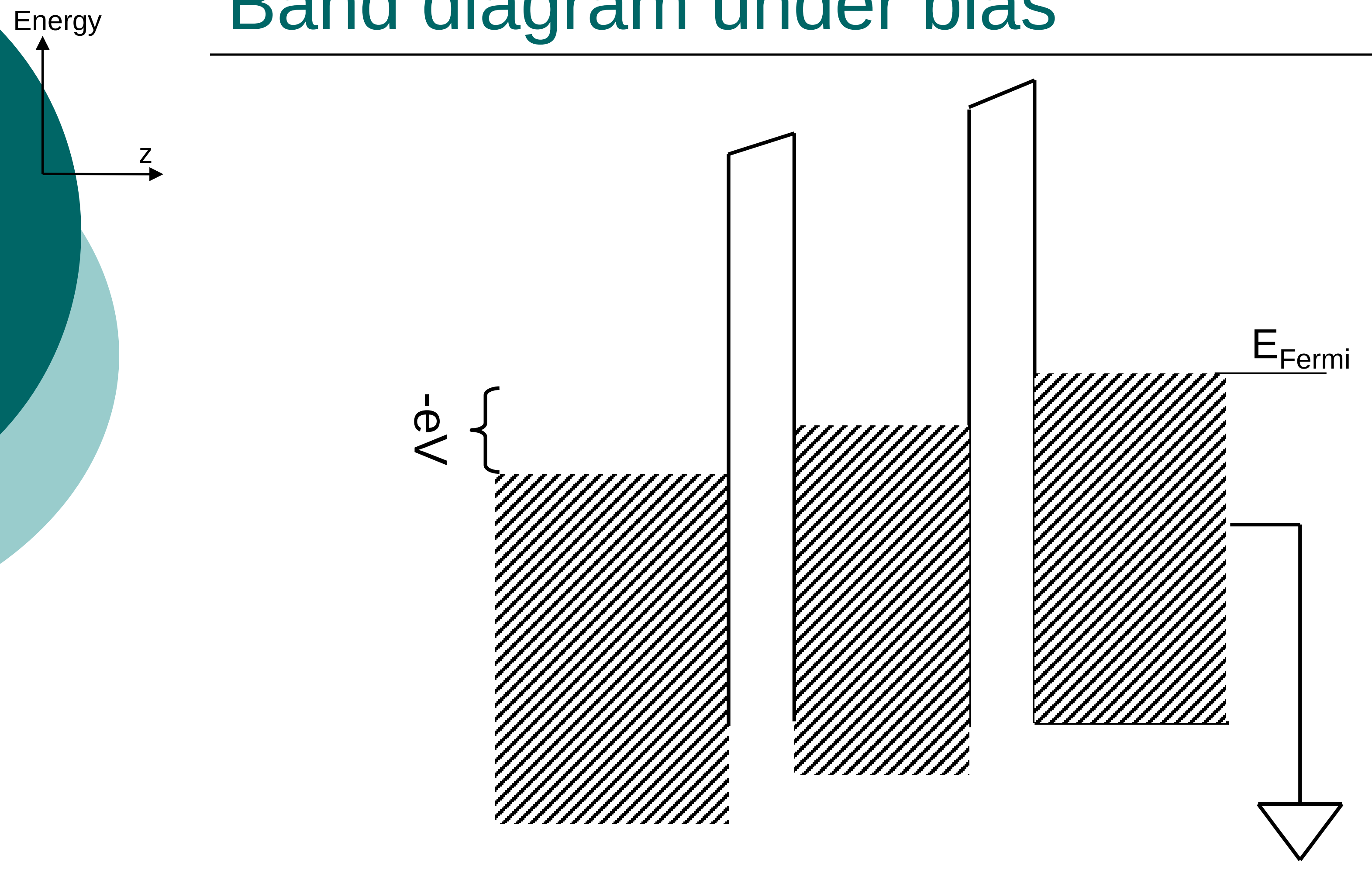




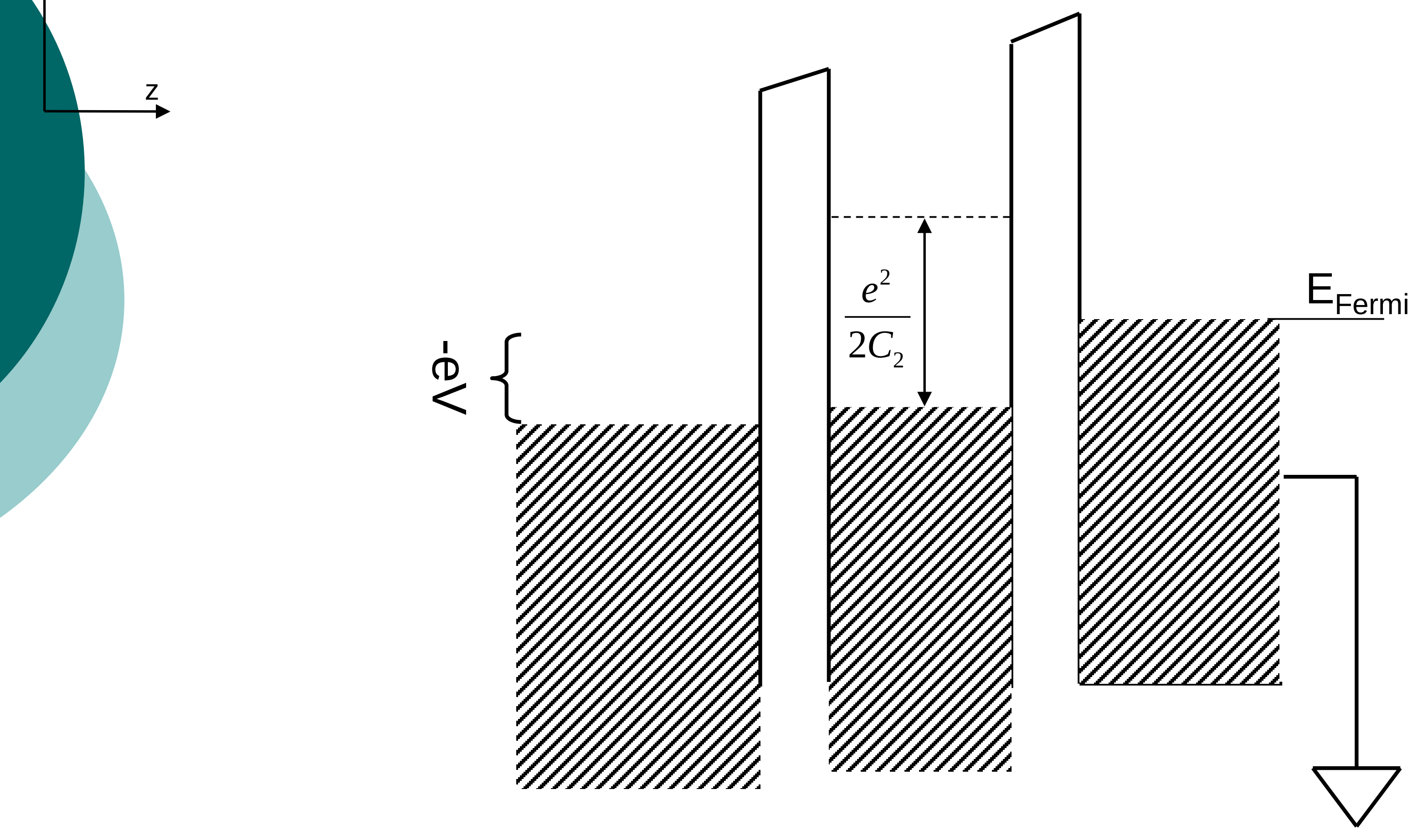
Band diagram with Coulomb "gap"



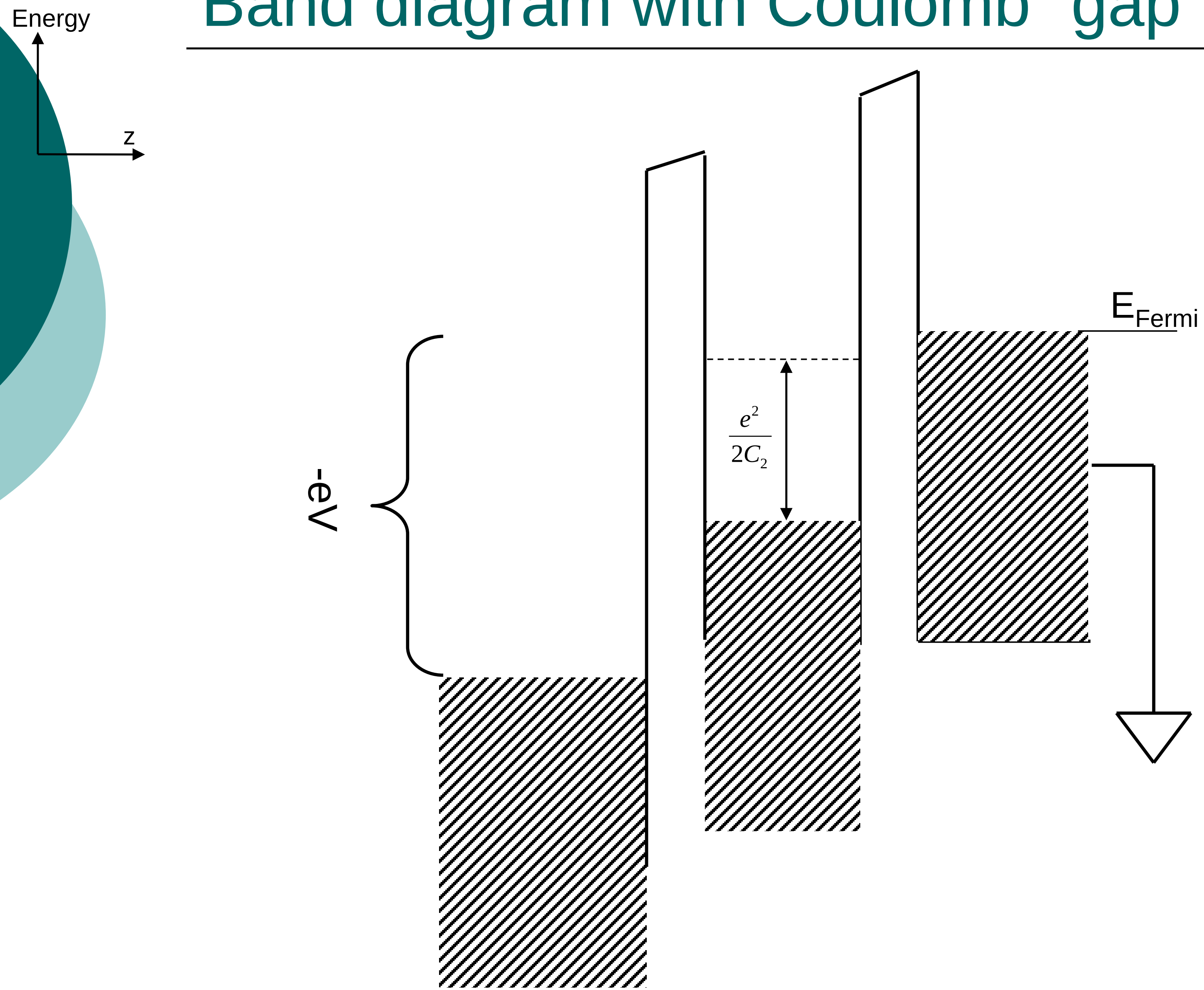
Band diagram under bias



Band diagram with Coulomb "gap"

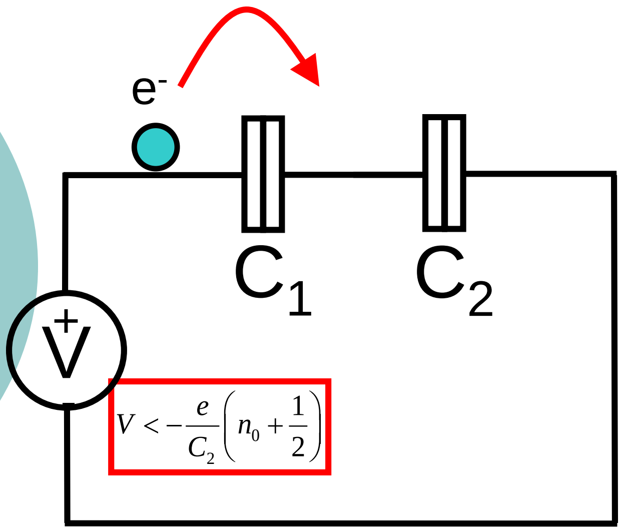


Band diagram with Coulomb "gap"



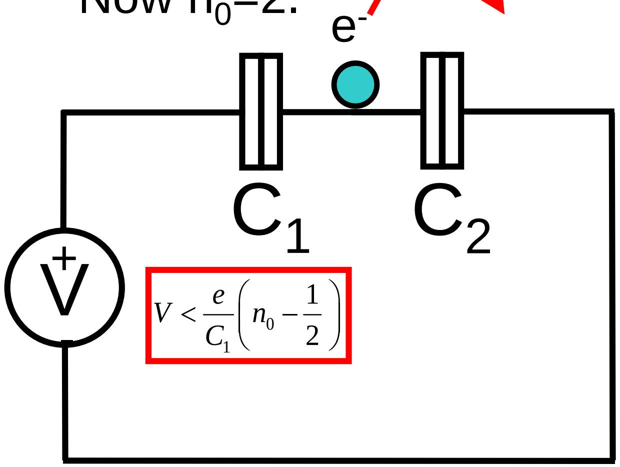
Higher voltages

Let n_0=1. Let C_1= C_2



$$V < -\frac{3e}{2C_2}$$

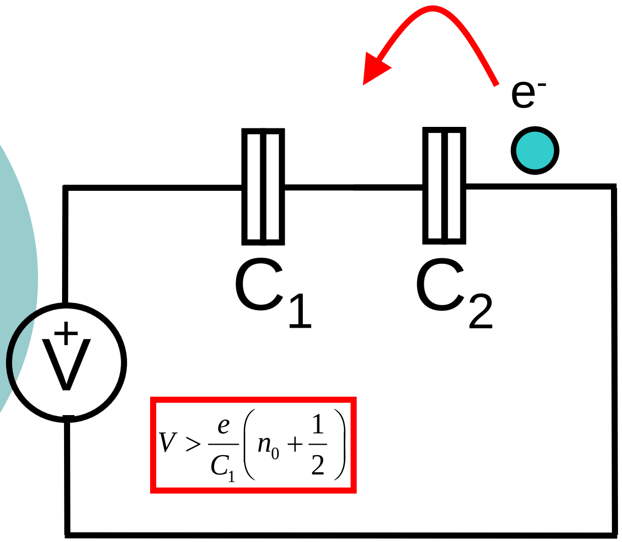
Now n_0=2.



$$V < -\frac{3e}{2C_2}$$

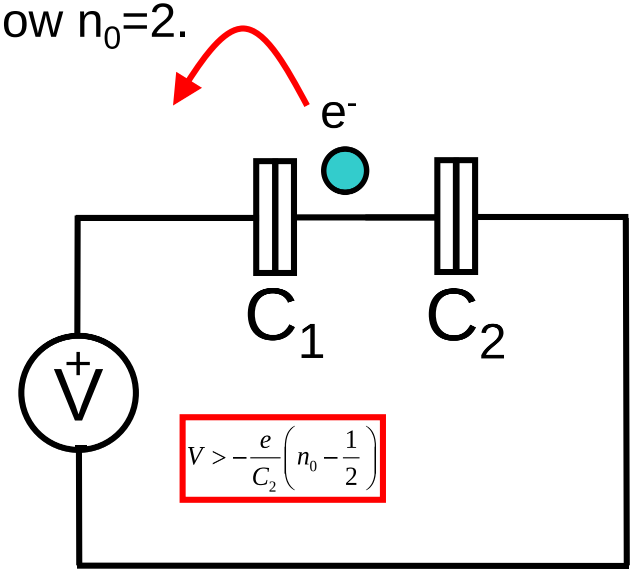
Higher voltages

Let $n_0=1$. Let $C_1= C_2$



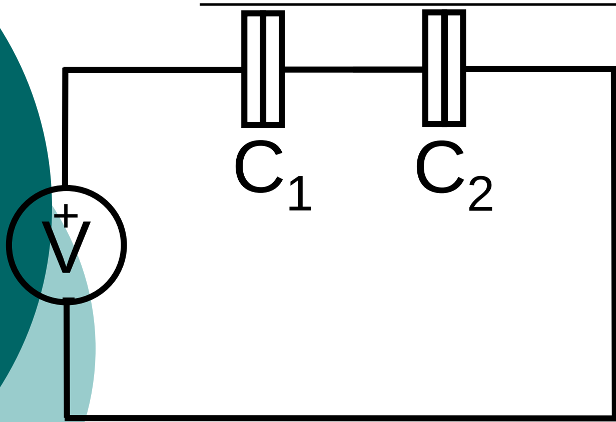
$$V > \frac{3e}{2C_2}$$

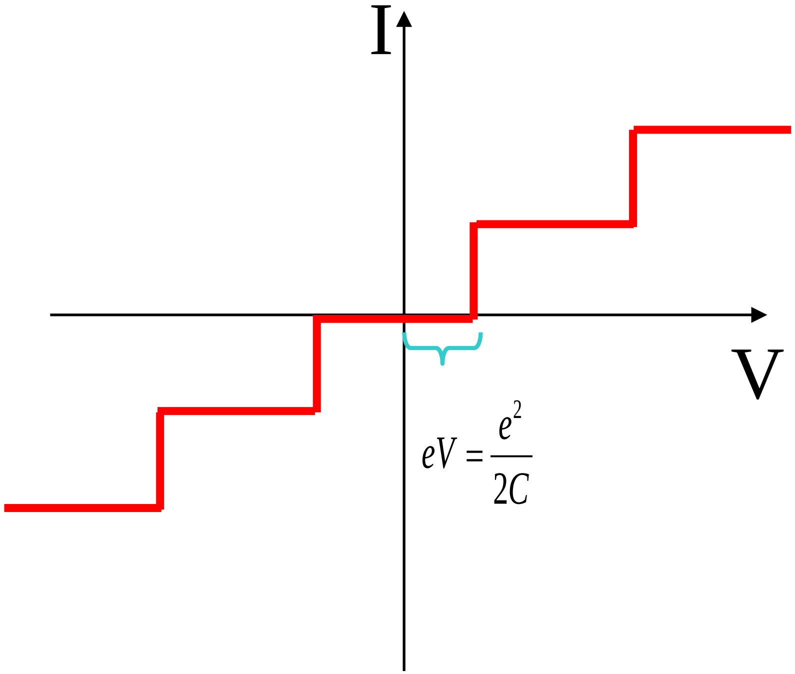
Now $n_0=2$.



$$V > \frac{3e}{2C_2}$$

Coulomb staircase





Overall slope is R_{tunnel}, which is *large*.