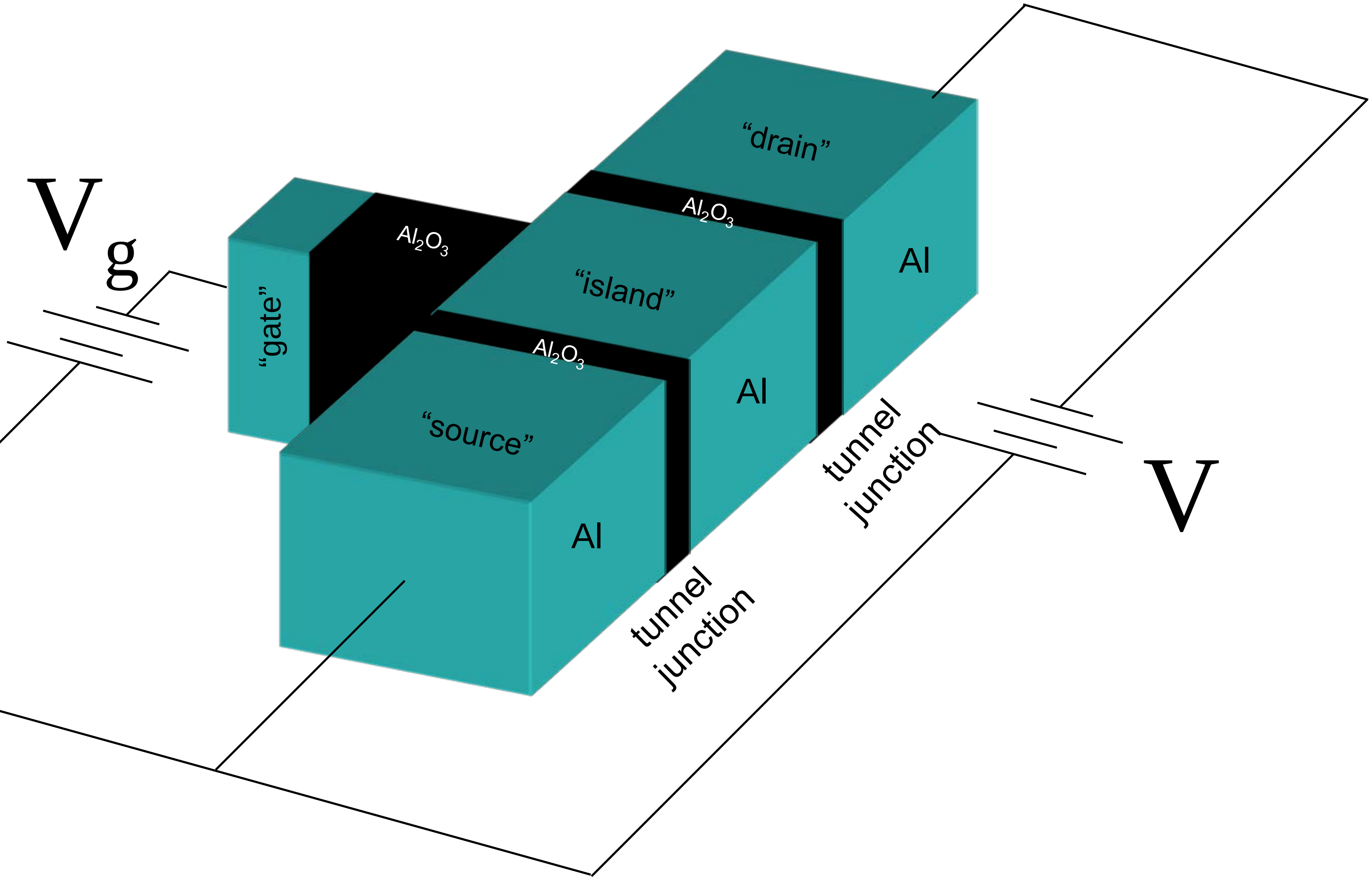
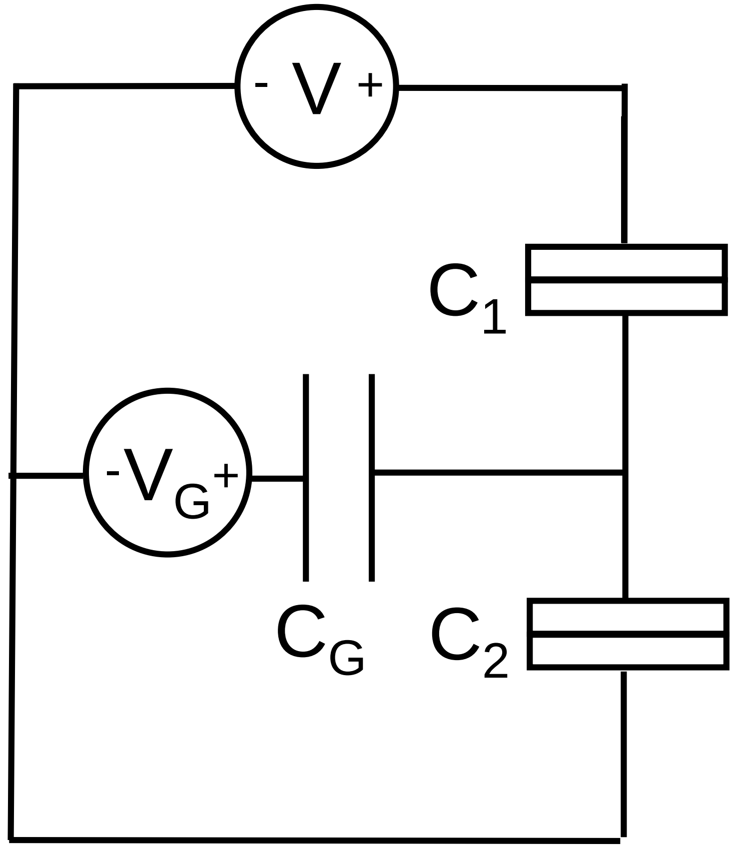
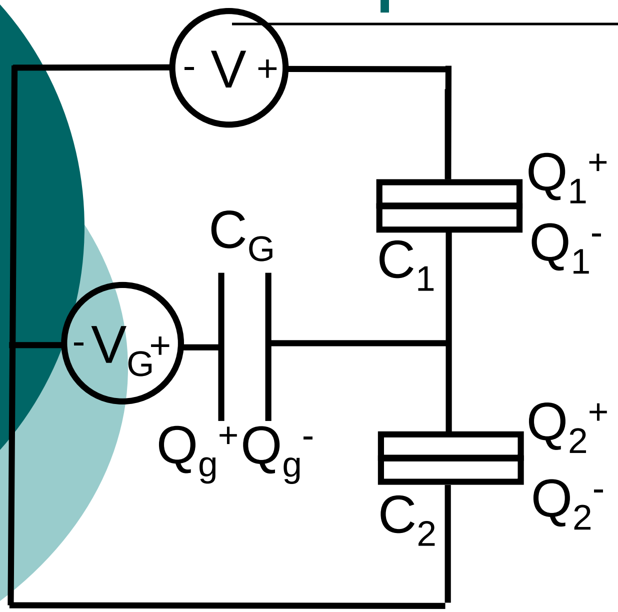
Lecture 8: Single electron transistor



Single electron transistor circuit



Capacitor charges



Kirchoff:
$$V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$V_g = \frac{Q_g}{C_g} + \frac{Q_2}{C_2}$$

Island charge: $Q_i = Q_2 - Q_1 - Q_g$

3 equations, 3 unknowns, solve:

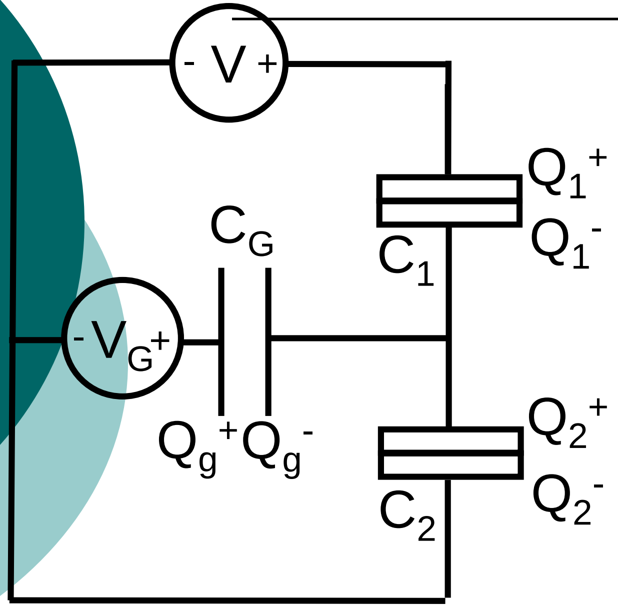
$$Q_1 = \frac{C_1 V (C_2 + C_g) - C_1 C_g (Q_i + V_g)}{C_\Sigma}$$

$$C_\Sigma \equiv C_1 + C_2 + C_g$$

$$Q_2 = \frac{C_2 Q_i + C_1 C_2 V + C_g C_2 V_g}{C_\Sigma}$$

$$Q_2 = \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma}$$

Electrostatic energy



$$E = \frac{Q_1^{\ 2}}{2C_1} + \frac{Q_2^{\ 2}}{2C_2} + \frac{Q_G^{\ 2}}{2C_G}$$

$$Q_1 = \frac{C_1 V (C_2 + C_g) - C_1 C_G V_G - C_1 Q_i}{C_\Sigma}$$

$$Q_2 = \frac{C_2 Q_i + C_1 C_2 V + C_g C_2 V_g}{C_\Sigma}$$

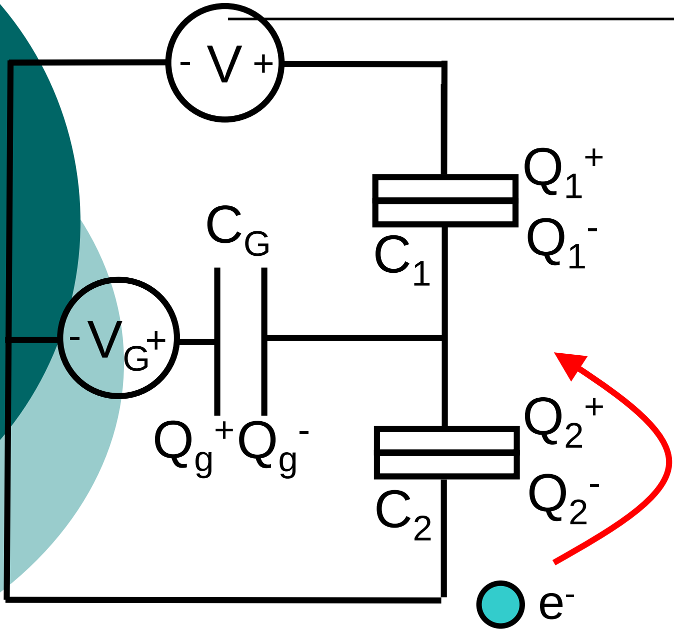
$$Q_G = \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma}$$

$$E = \frac{1}{2C_\Sigma}\left[C_G C_1 + \left(V - V_G\right)^2 + C_1 C_2 V^2 + C_G C_2 V_G^2 + Q_i^2 \right]$$

Free energy :

$$G = E - Q_1 V - Q_G V_G$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before:

$$Q_i = -n_0 e$$

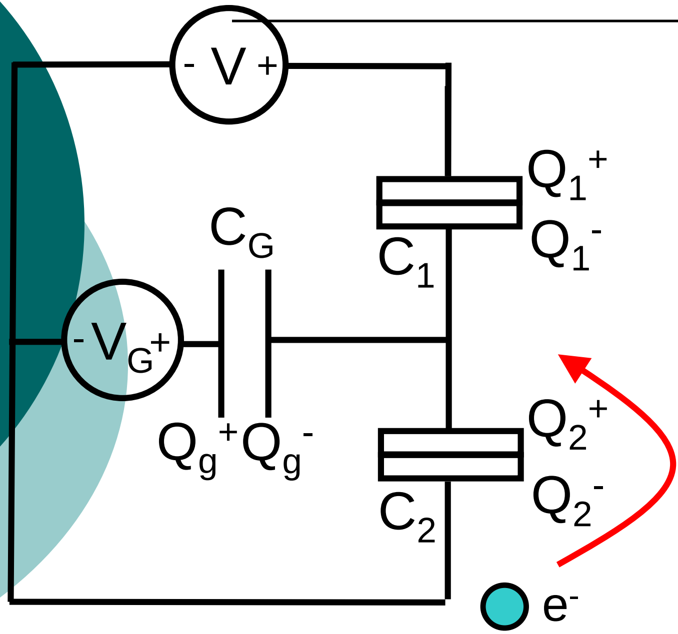
After:

$$Q_i = -n_0 e - e$$

$$E = \frac{1}{2C_\Sigma}\left[C_G C_1 + \left(V - V_G\right)^2 + C_1 C_2 V^2 + C_G C_2 V_G^2 + Q_i^2\right]$$

$$\Delta E = \frac{-2n_0 e^2 - e^2}{2C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before: $Q_i = -n_0 e$

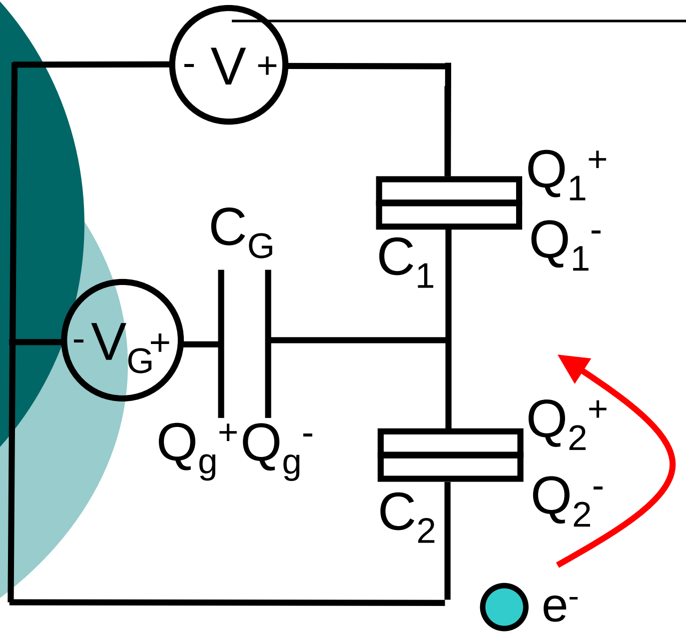
After: $Q_i = -n_0 e - e$

$$Q_1 = \frac{C_1 V(C_2 + C_g) - C_1 C_G V_G - C_1 Q_i}{C_\Sigma}$$

$$\Delta Q_1 = \Delta \frac{C_1 V(C_2 + C_g) - C_1 C_G V_G - C_1 Q_i}{C_\Sigma} = \frac{-C_1\left(-n_0 e\right)}{C_\Sigma} - \frac{-C_1\left(-n_0 e - e\right)}{C_\Sigma}$$

$$\Delta Q_1 = \frac{-eC_1}{C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before: $Q_i = -n_0 e$

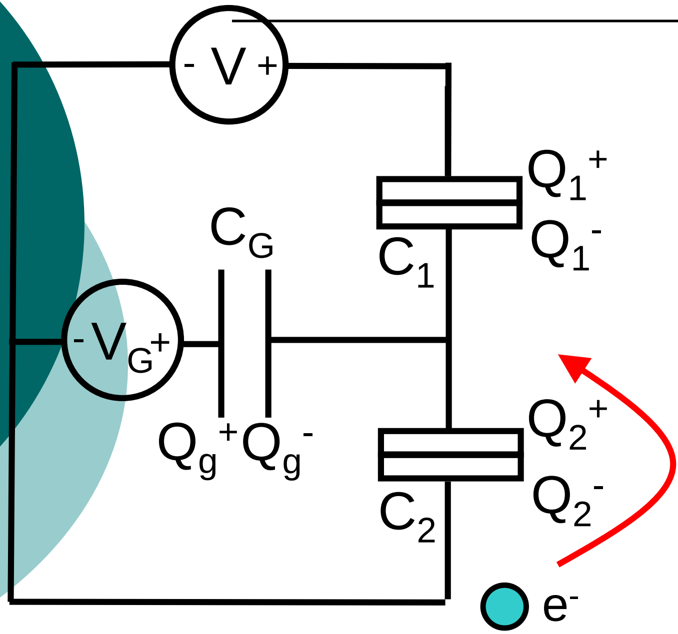
After: $Q_i = -n_0 e - e$

$$Q_G = \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma}$$

$$\Delta Q_G = \Delta \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma} = \frac{-C_g\left(-n_0 e\right)}{C_\Sigma} - \frac{-C_g\left(-n_0 e - e\right)}{C_\Sigma}$$

$$\Delta Q_G = \frac{-eC_g}{C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

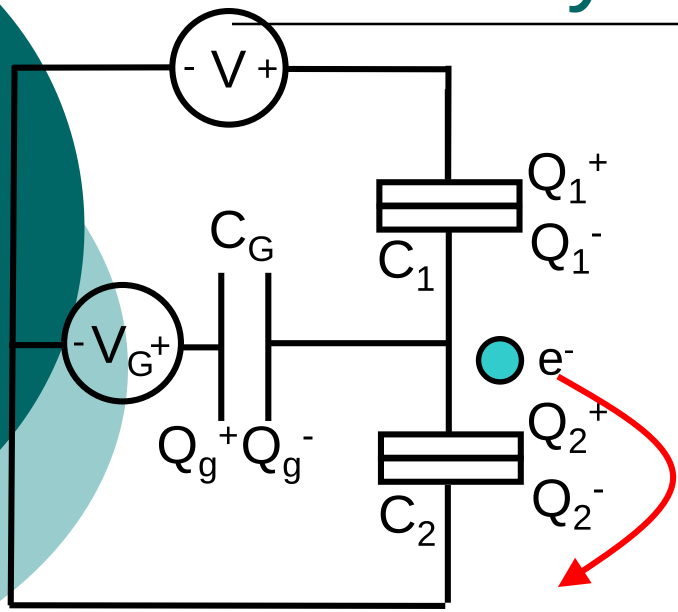
$$\Delta E = \frac{-2n_0 e^2 - e^2}{2C_\Sigma}$$

$$\Delta Q_1 = \frac{-eC_1}{C_\Sigma} \qquad \Delta Q_G = \frac{-eC_g}{C_\Sigma}$$

$$\Delta G = \frac{-2n_0 e^2 - e^2}{2C_\Sigma} + V\frac{eC_1 C_g}{C_\Sigma} + V_G \frac{eC_g}{C_\Sigma}$$

$$\Delta G = \frac{e}{C_\Sigma}\left[-n_0 e - \frac{e}{2} + C_1 V + C_g V_G\right] > 0$$

Similarly

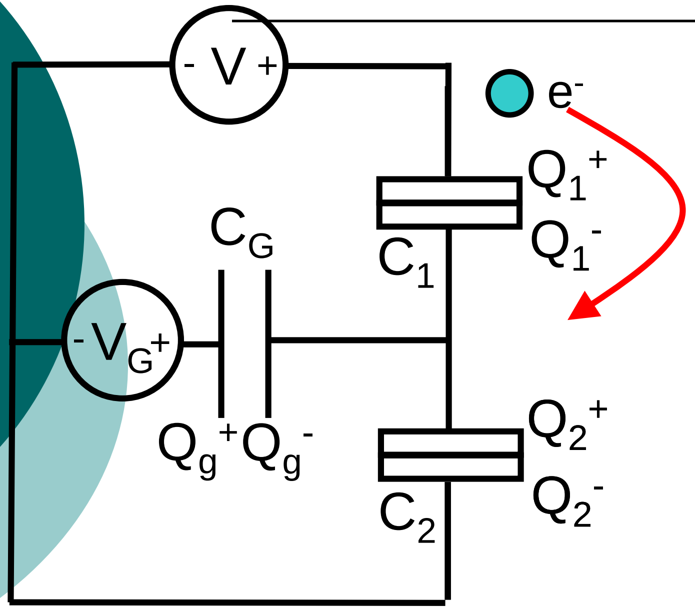


Allowed only if:

$$\Delta G = \frac{e}{C_\Sigma}\left[+n_0 e - \frac{e}{2} - C_1 V - C_g V_G \right] > 0$$

n_0 is the number of electrons on the island *before* the tunnel event.

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before:

$$Q_i = -n_0 e$$

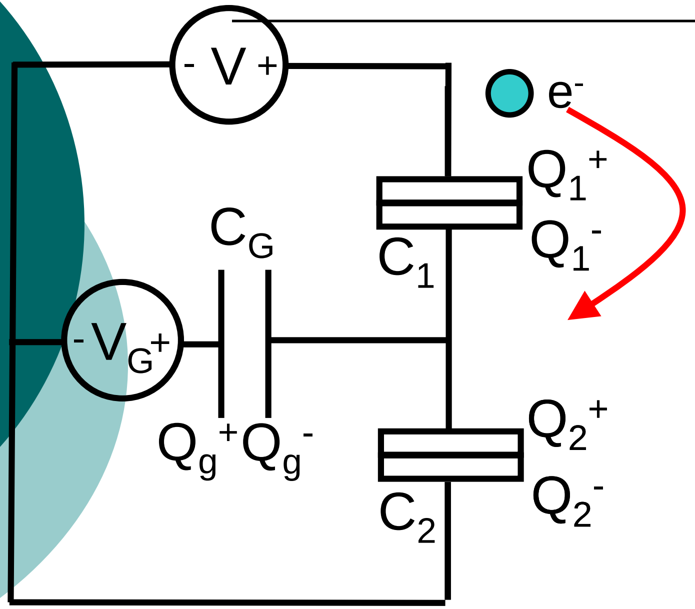
After:

$$Q_i = -n_0 e + e$$

$$E = \frac{1}{2C_\Sigma}\left[C_G C_1 + (V - V_G)^2 + C_1 C_2 V^2 + C_G C_2 V_G^2 + Q_i^2\right]$$

$$\Delta E = \frac{-2n_0 e^2 - e^2}{2C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before:

$$Q_i = -n_0 e$$

After:

$$Q_i = -n_0 e + e$$

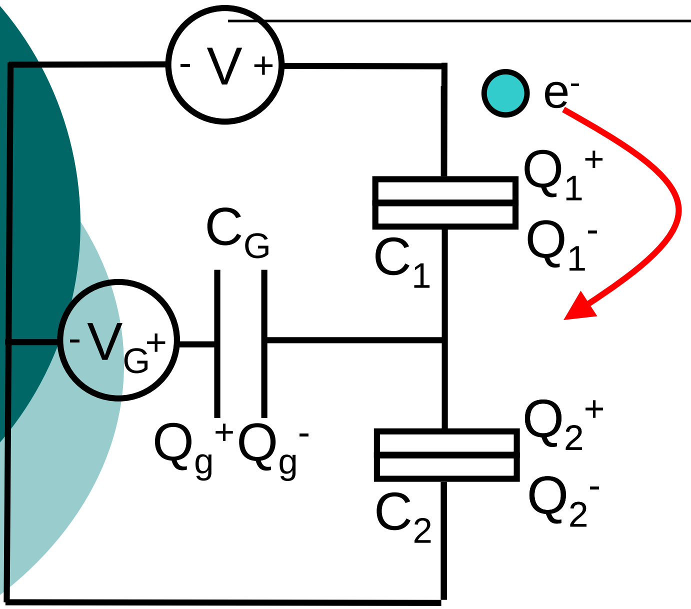
$$Q_1 = \frac{C_1 V(C_2 + C_g) - C_1 C_G V_G - C_1 Q_i}{C_\Sigma}$$

$$\Delta Q_1 = \Delta \frac{C_1 V(C_2 + C_g) - C_1 C_G V_G - C_1 Q_i}{C_\Sigma} = \frac{-C_1(-n_0 e)}{C_\Sigma} - \frac{-C_1(-n_0 e - e)}{C_\Sigma}$$

$$\Delta Q_{1,polarization} = \frac{-eC_1}{C_\Sigma} \qquad \Delta Q_{1,tunnel} = e$$

$$\Delta Q_{1,total} = \Delta Q_{1,polarization} + \Delta Q_{1,tunnel} = \frac{-eC_1}{C_\Sigma} + e = \frac{-eC_1}{C_1 + C_2 + C_G} + \frac{eC_1 + eC_2 + eC_G}{C_1 + C_2 + C_G} = e\frac{C_2 + C_G}{C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before:

$$Q_i = -n_0 e$$

After:

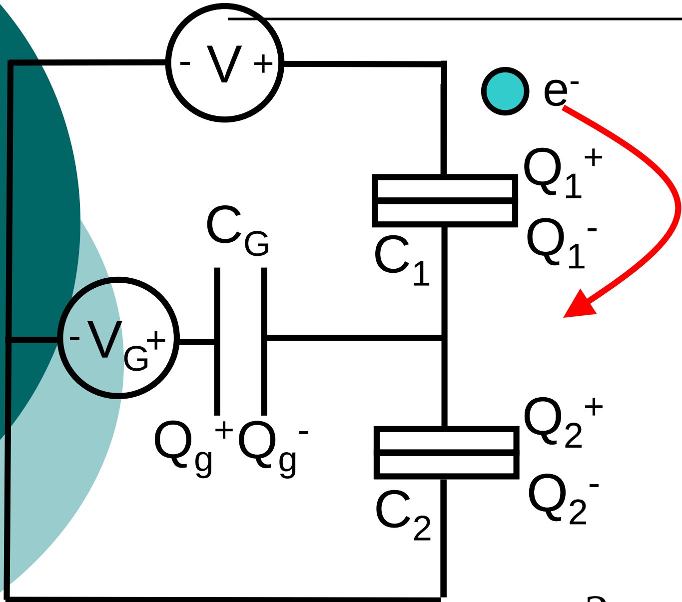
$$Q_i = -n_0 e + e$$

$$Q_G = \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma}$$

$$\Delta Q_G = \Delta \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma} = \frac{-C_g\left(-n_0 e\right)}{C_\Sigma} - \frac{-C_g\left(-n_0 e - e\right)}{C_\Sigma}$$

$$\Delta Q_G = \frac{-eC_g}{C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

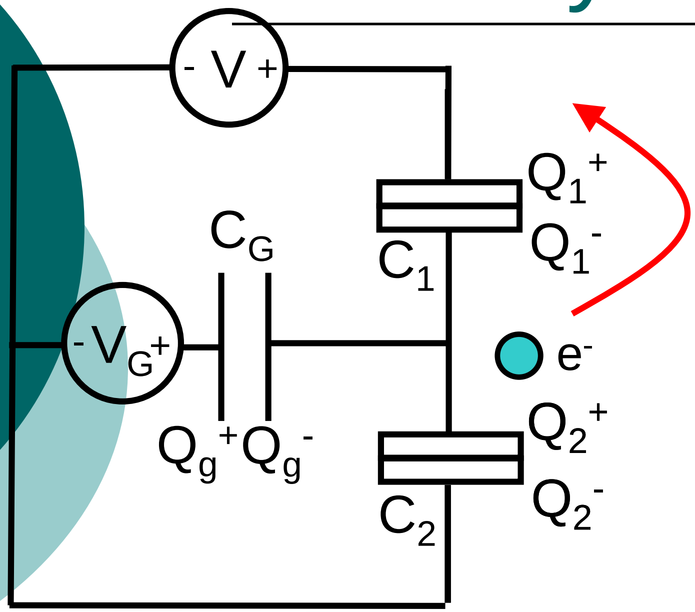
$$\Delta E = \frac{-2n_0 e^2 - e^2}{2C_\Sigma}$$

$$\Delta Q_{1,total} = e\frac{C_2 + C_G}{C_\Sigma} \qquad \Delta Q_G = \frac{-eC_g}{C_\Sigma}$$

$$\Delta G = \frac{-2n_0 e^2 - e^2}{2C_\Sigma} - Ve\frac{C_2 + C_G}{C_\Sigma} + V_G \frac{eC_g}{C_\Sigma}$$

$$\Delta G = \frac{e}{C_\Sigma}\left[-n_0 e - \frac{e}{2} - V\left(C_2 + C_G\right) + V_G C_g\right] > 0$$

Similarly:



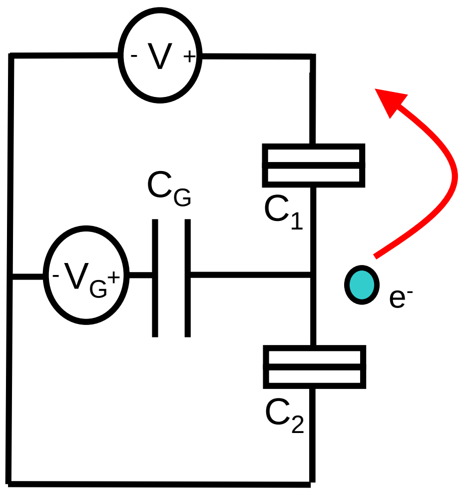
Allowed only if:

$$\Delta G = \frac{e}{C_\Sigma}\left[+n_0 e - \frac{e}{2} + V\left(C_2 + C_G\right) - V_G C_g \right] > 0$$

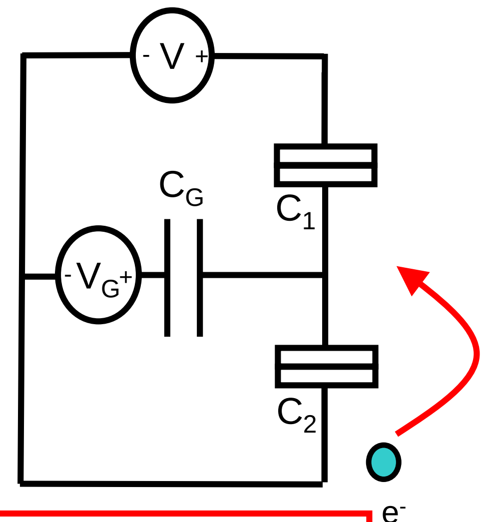
n_0 is the number of electrons on the island *before* the tunnel event.

Summary:

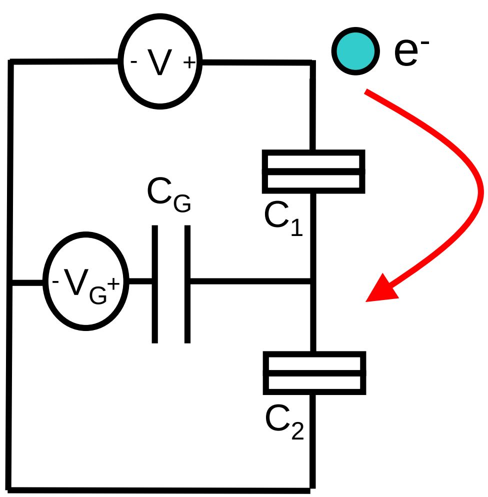
$$+n_0 e - \frac{e}{2} + V\left(C_2 + C_G\right) - V_G C_g > 0$$



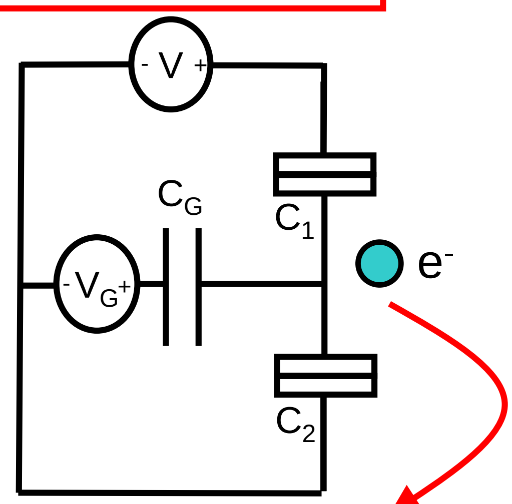
$$-n_0 e - \frac{e}{2} + C_1 V + C_g V_G > 0$$



$$-n_0 e - \frac{e}{2} - V\left(C_2 + C_G\right) + V_G C_g > 0$$

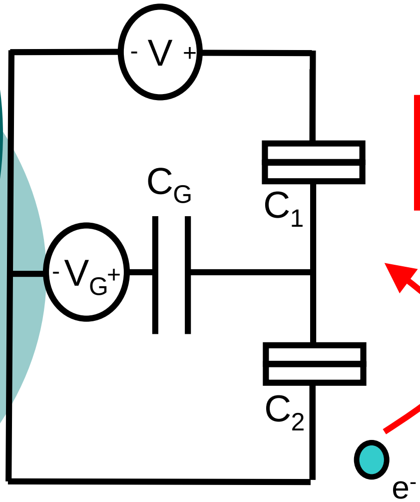


$$+n_0 e - \frac{e}{2} - C_1 V - C_g V_G > 0$$



Current?

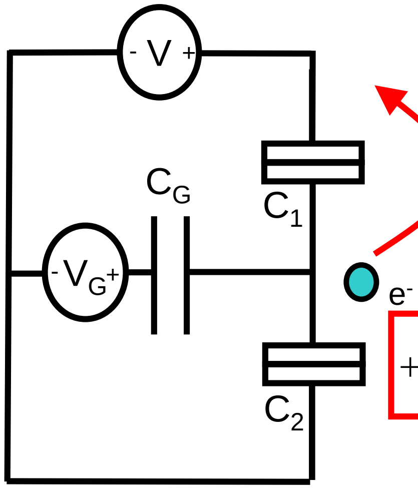
Let n_0=0.



$$-\frac{e}{2}+C_1V+C_gV_G>0$$

$$V>\frac{e}{2C_1}-\frac{C_g}{C_1}V_G$$

Now n_0=1.

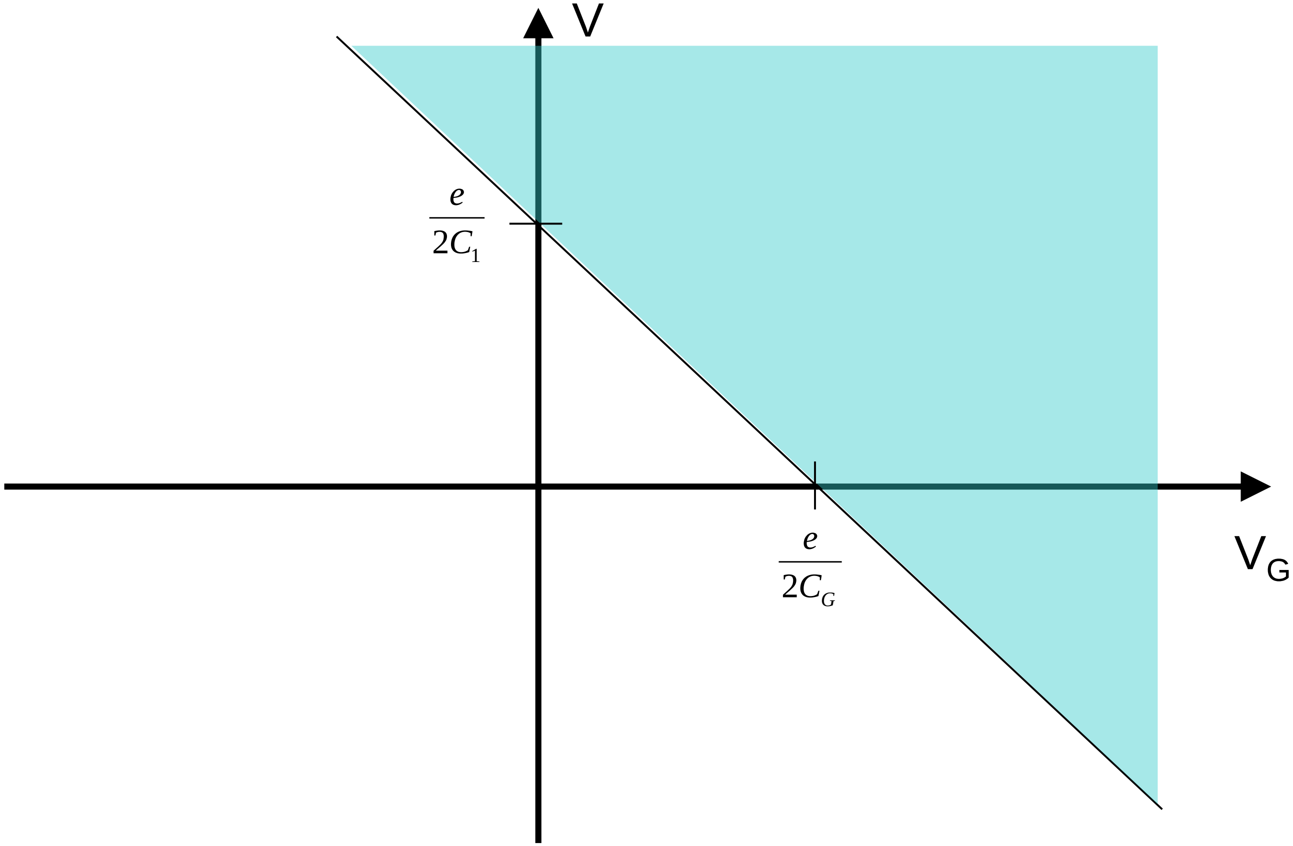


$$+\frac{e}{2}+V\left(C_2+C_G\right)-V_GC_g>0$$

$$V>\frac{C_g}{\left(C_2+C_G\right)}V_G-\frac{e}{2\left(C_2+C_G\right)}$$

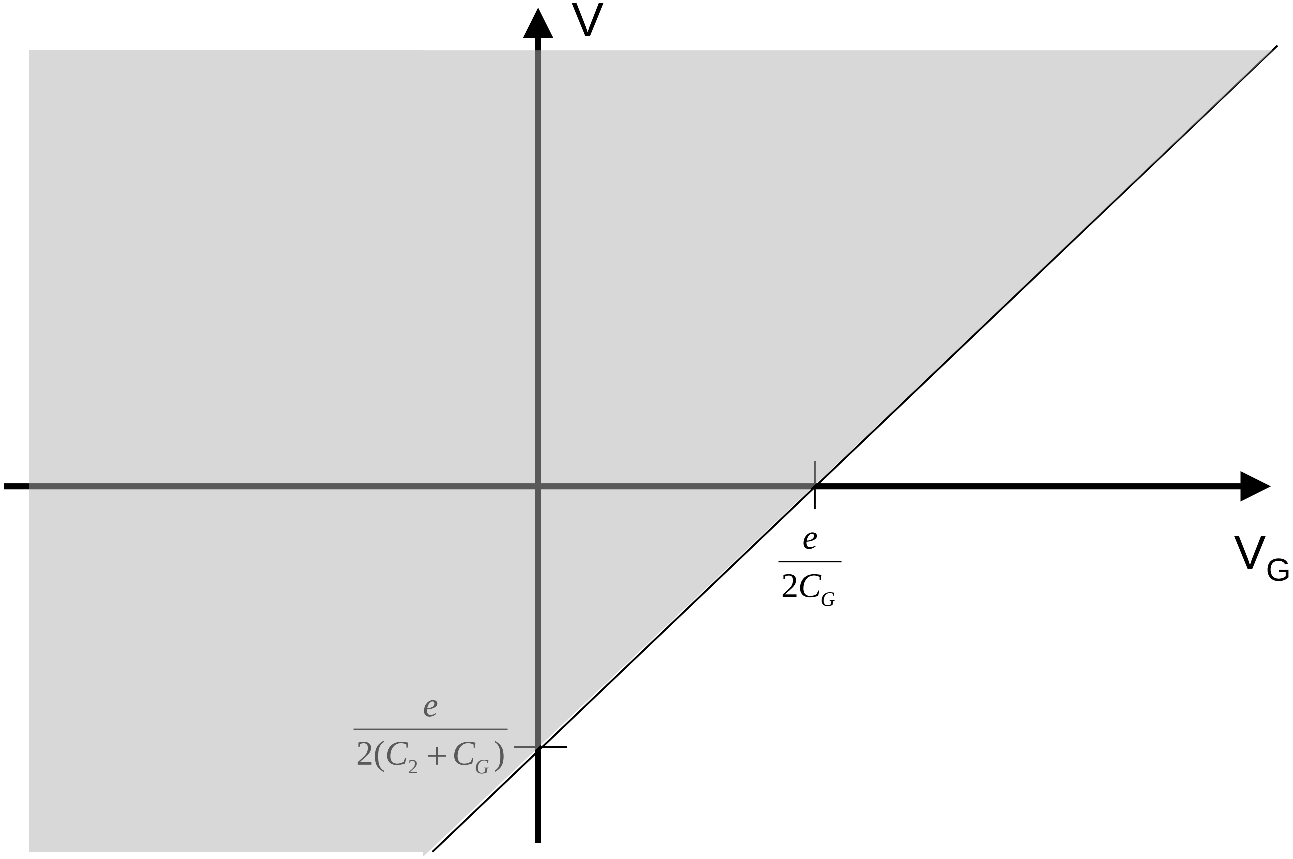
Current?

$$V > \frac{e}{2C_1} - \frac{C_g}{C_1} V_G$$



Current?

$$V > \frac{C_g}{\left(C_2 + C_G\right)} V_G - \frac{e}{2\left(C_2 + C_G\right)}$$

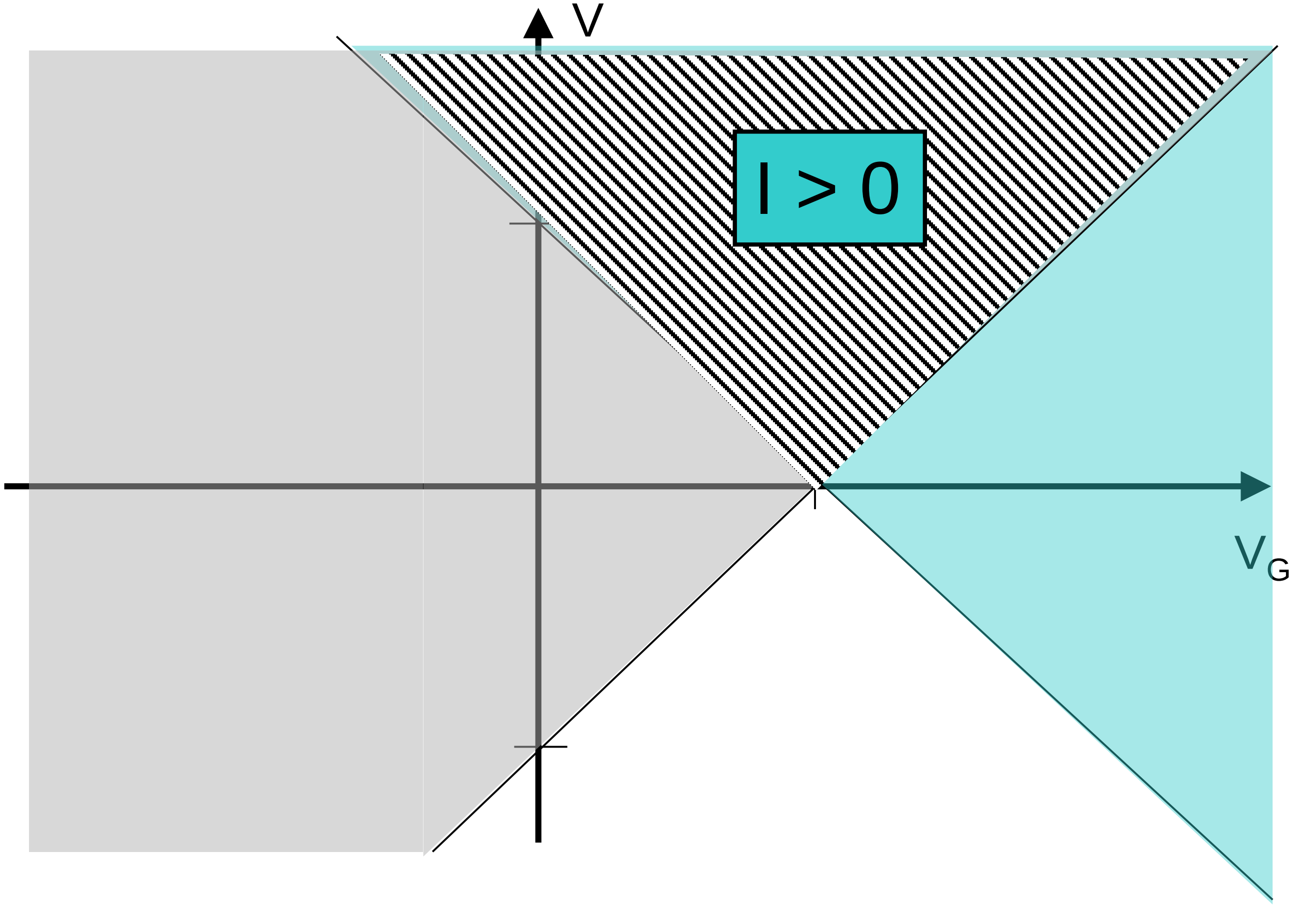


e⁻

Current?

$$V > \frac{e}{2C_1} - \frac{C_g}{C_1} V_G$$

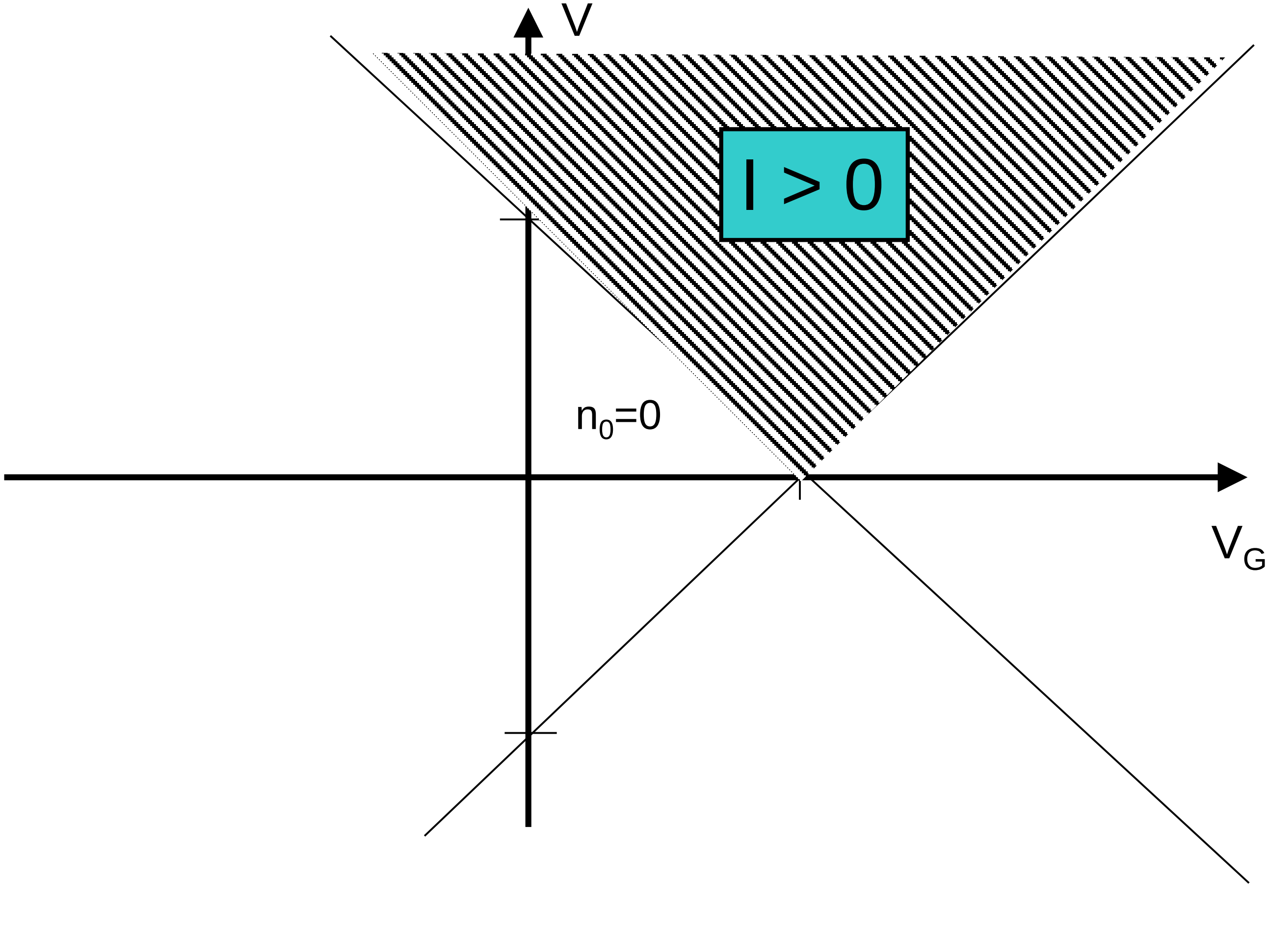
$$V > \frac{C_g}{(C_2 + C_G)} V_G - \frac{e}{2(C_2 + C_G)}$$



Current?

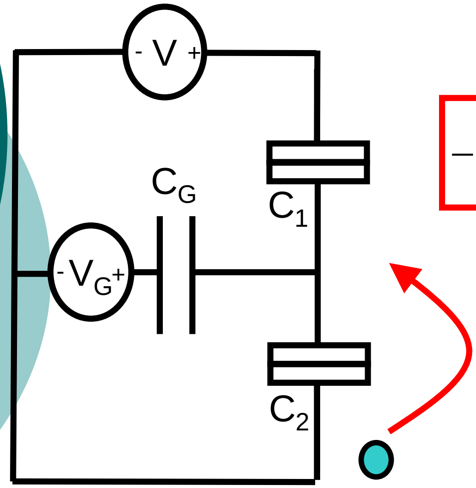
$$V > \frac{e}{2C_1} - \frac{C_g}{C_1} V_G$$

$$V > \frac{C_g}{\left(C_2 + C_G\right)} V_G - \frac{e}{2\left(C_2 + C_G\right)}$$



Current for 1 electrons on island:

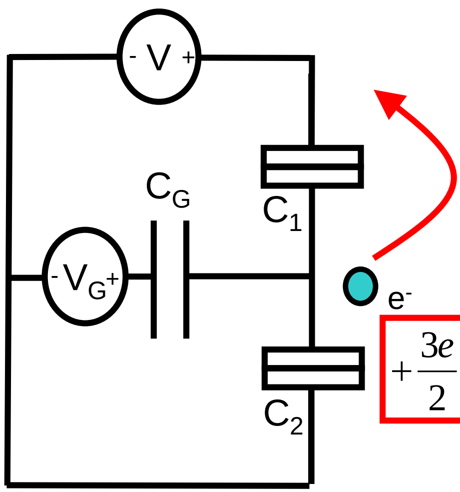
Let n_0=1.



$$-\frac{3e}{2}+C_1V+C_gV_G>0$$

$$V>\frac{3e}{2C_1}-\frac{C_g}{C_1}V_G$$

Now n_0=2.

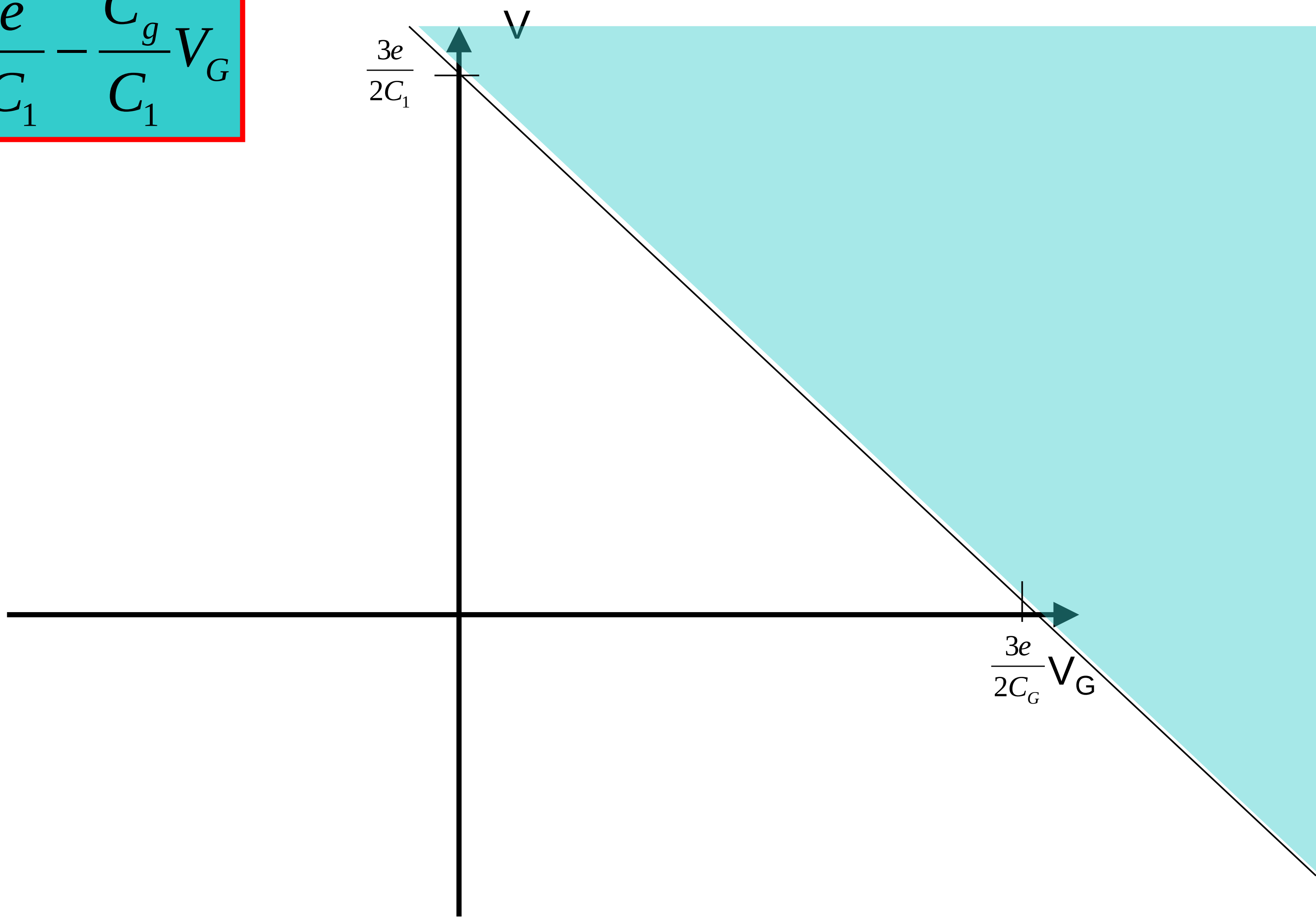


$$+\frac{3e}{2}+V\left(C_2+C_G\right)-V_GC_g>0$$

$$V>\frac{C_g}{\left(C_2+C_G\right)}V_G-\frac{3e}{2\left(C_2+C_G\right)}$$

Current?

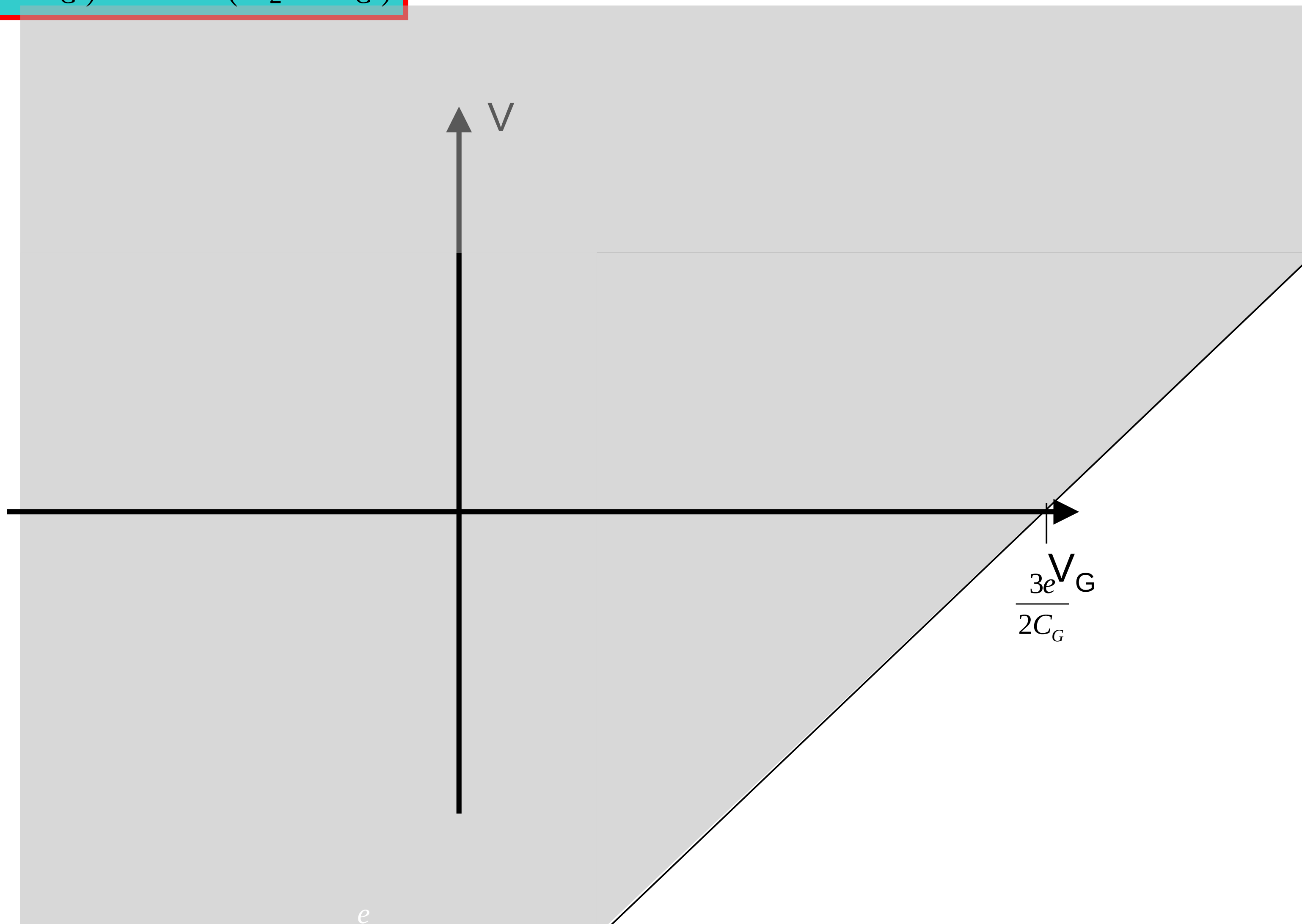
$$V > \frac{3e}{2C_1} - \frac{C_g}{C_1} V_G$$



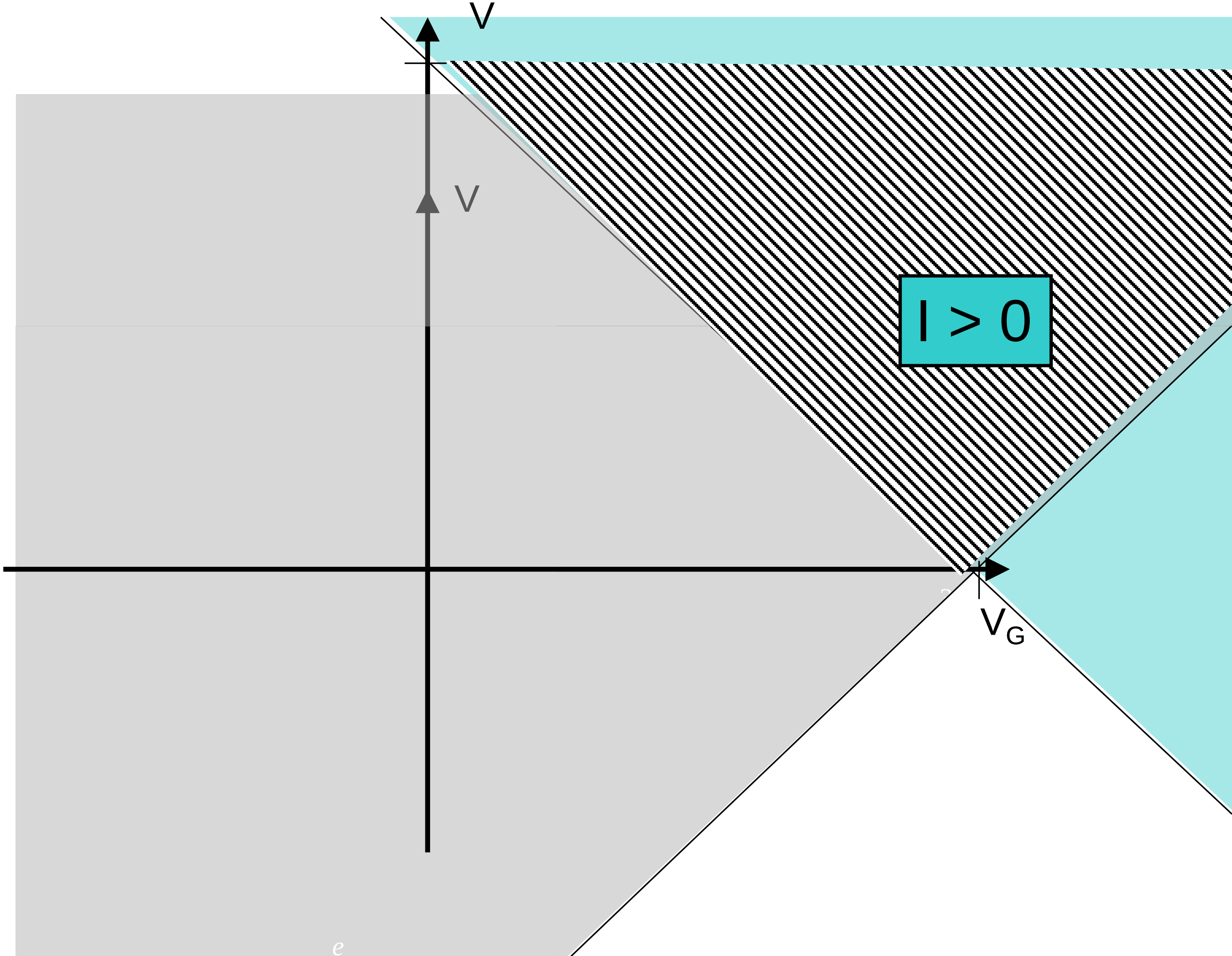
e-

Current?

$$V > \frac{C_g}{(C_2 + C_G)} V_G - \frac{3e}{2(C_2 + C_G)}$$

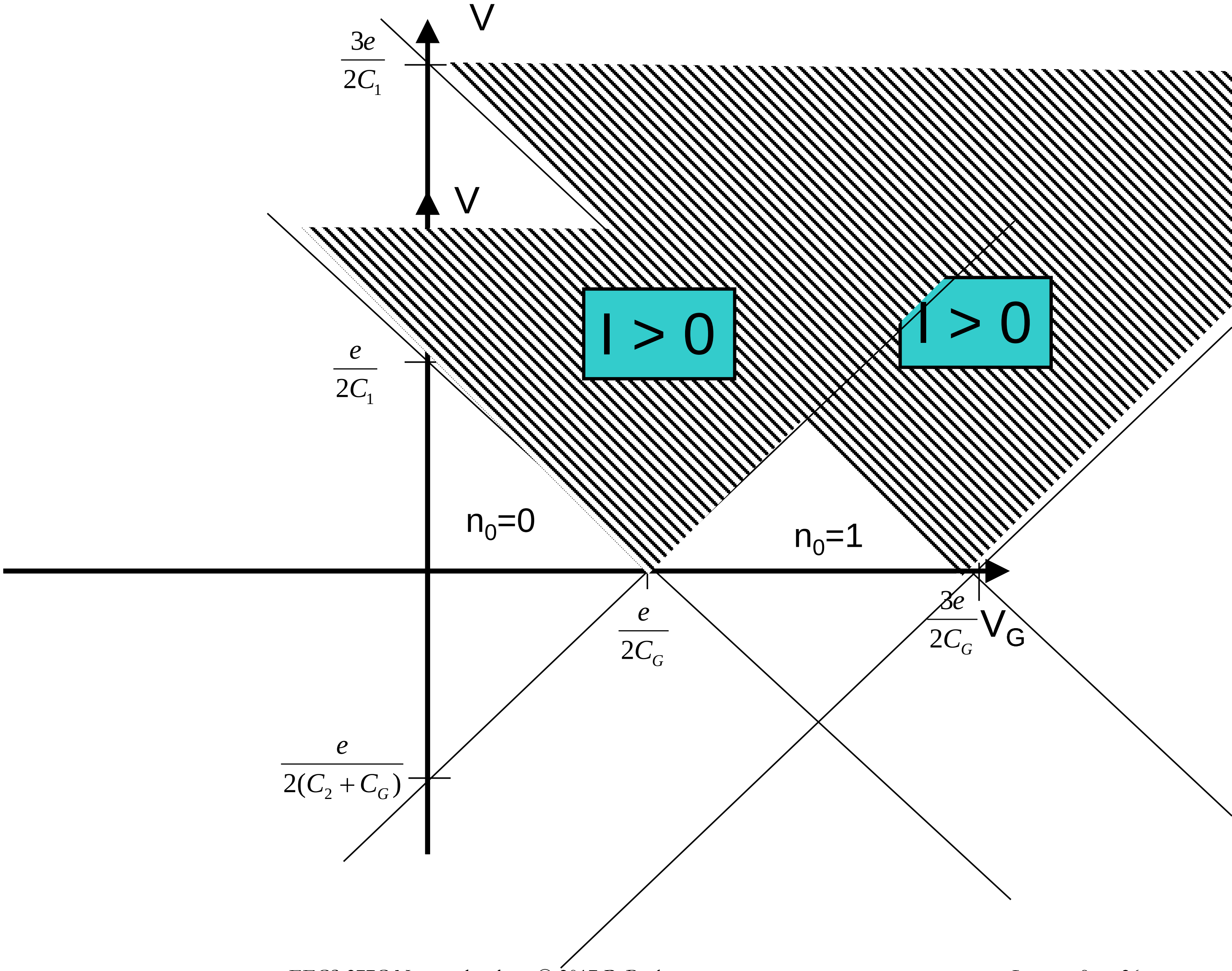


Current?

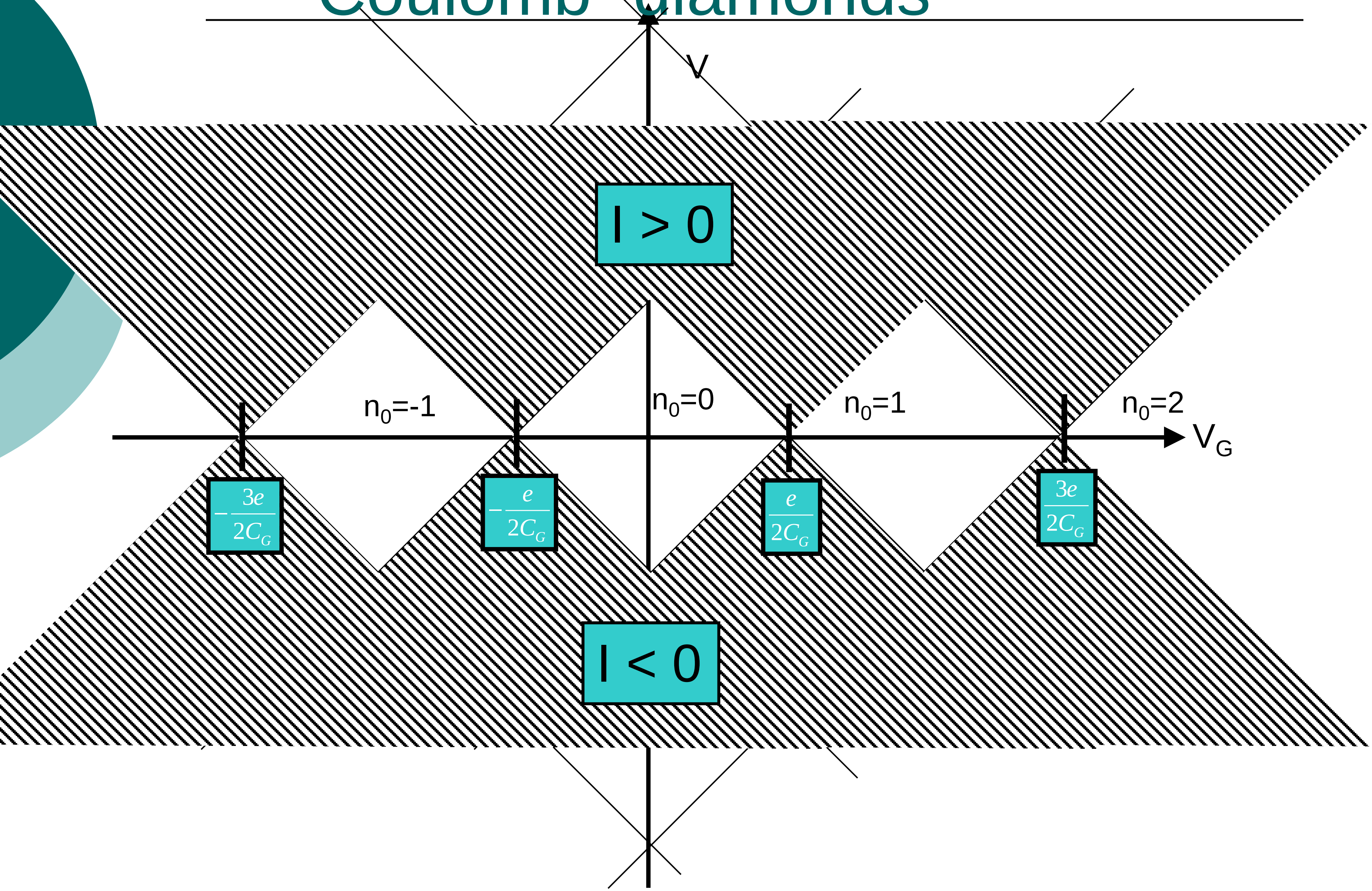


Current?

Current?



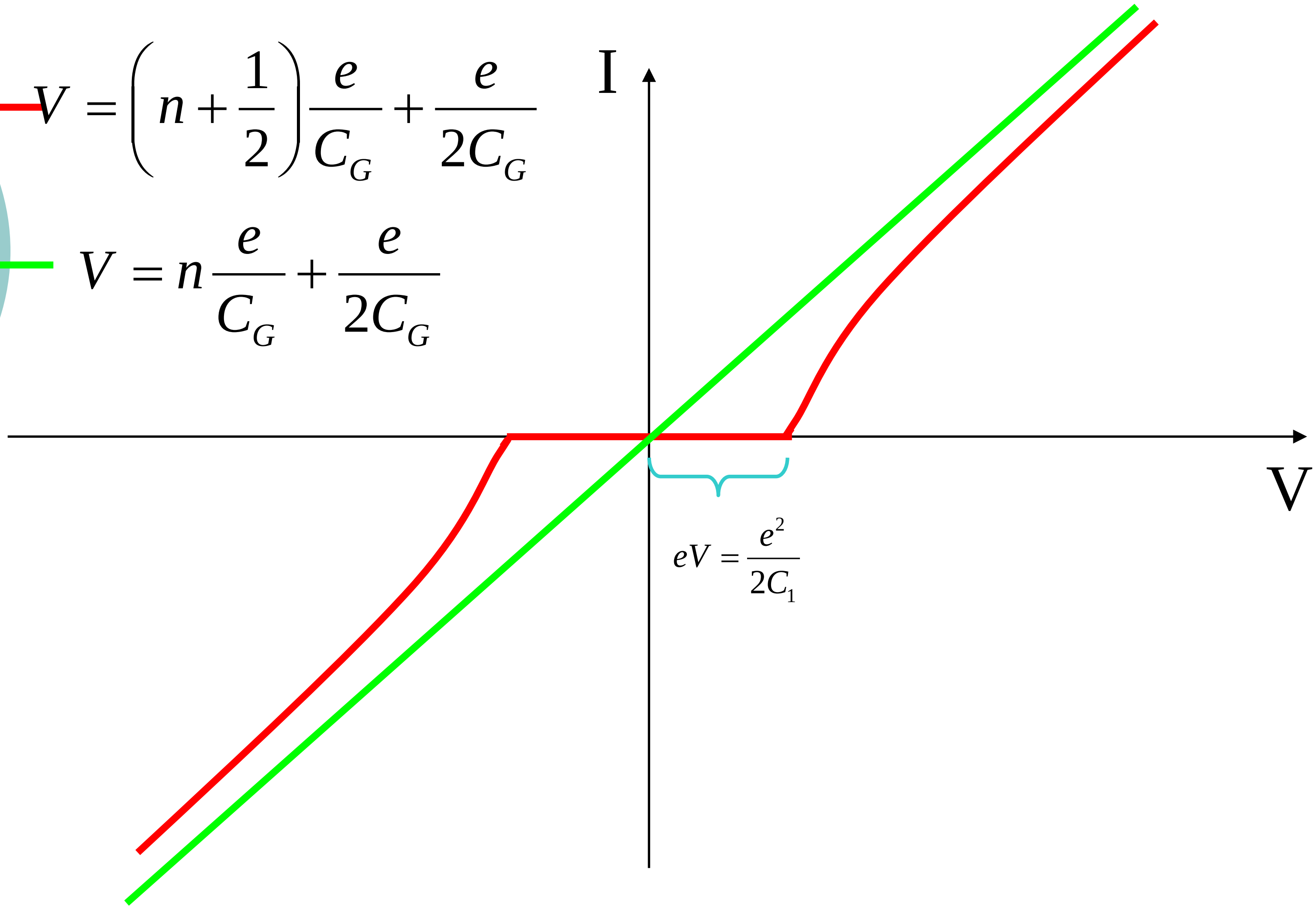
Coulomb “diamonds”



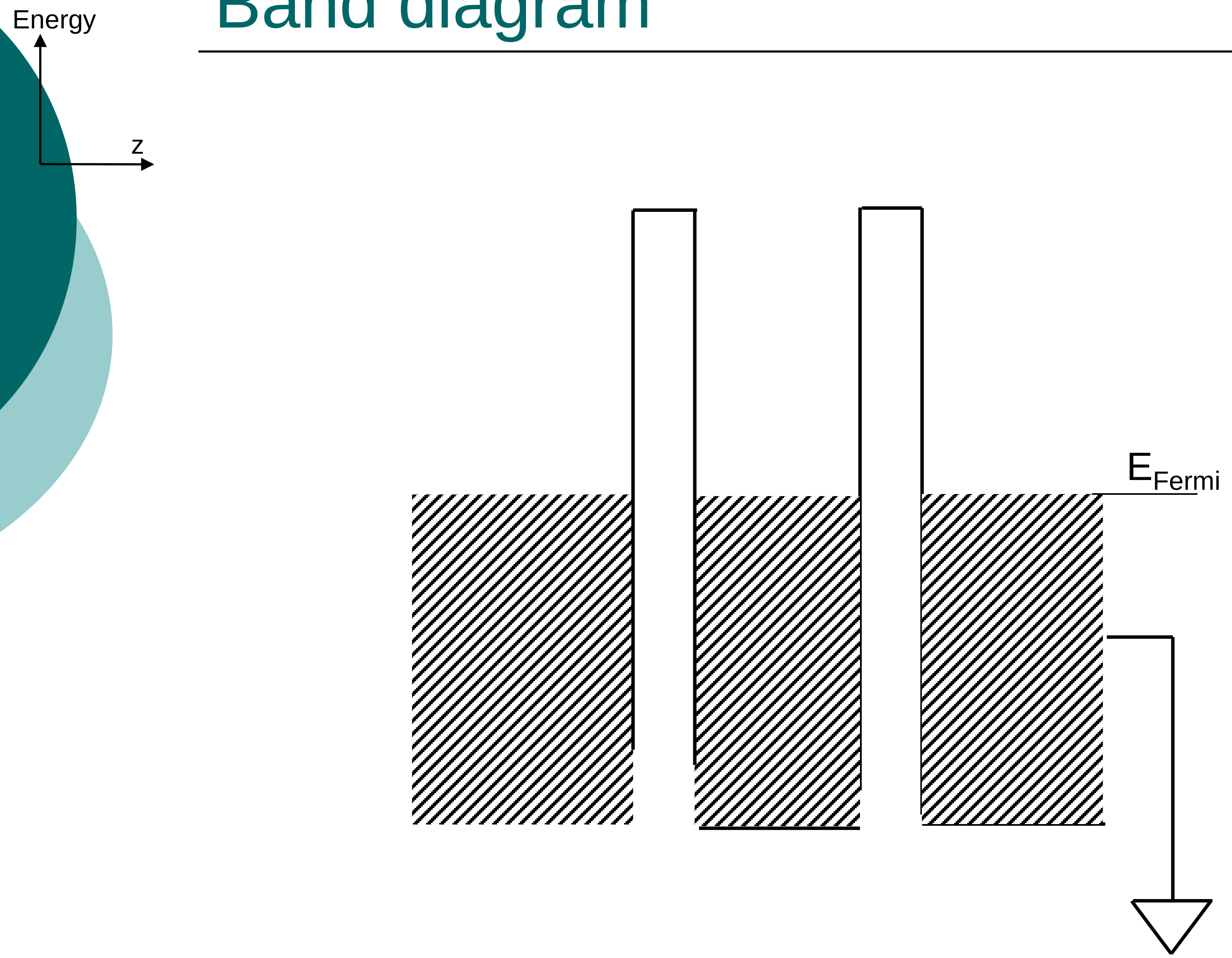
SET I-V curve vs. gate voltage

$$V = \left(n + \frac{1}{2}\right)\frac{e}{C_G} + \frac{e}{2C_G}$$

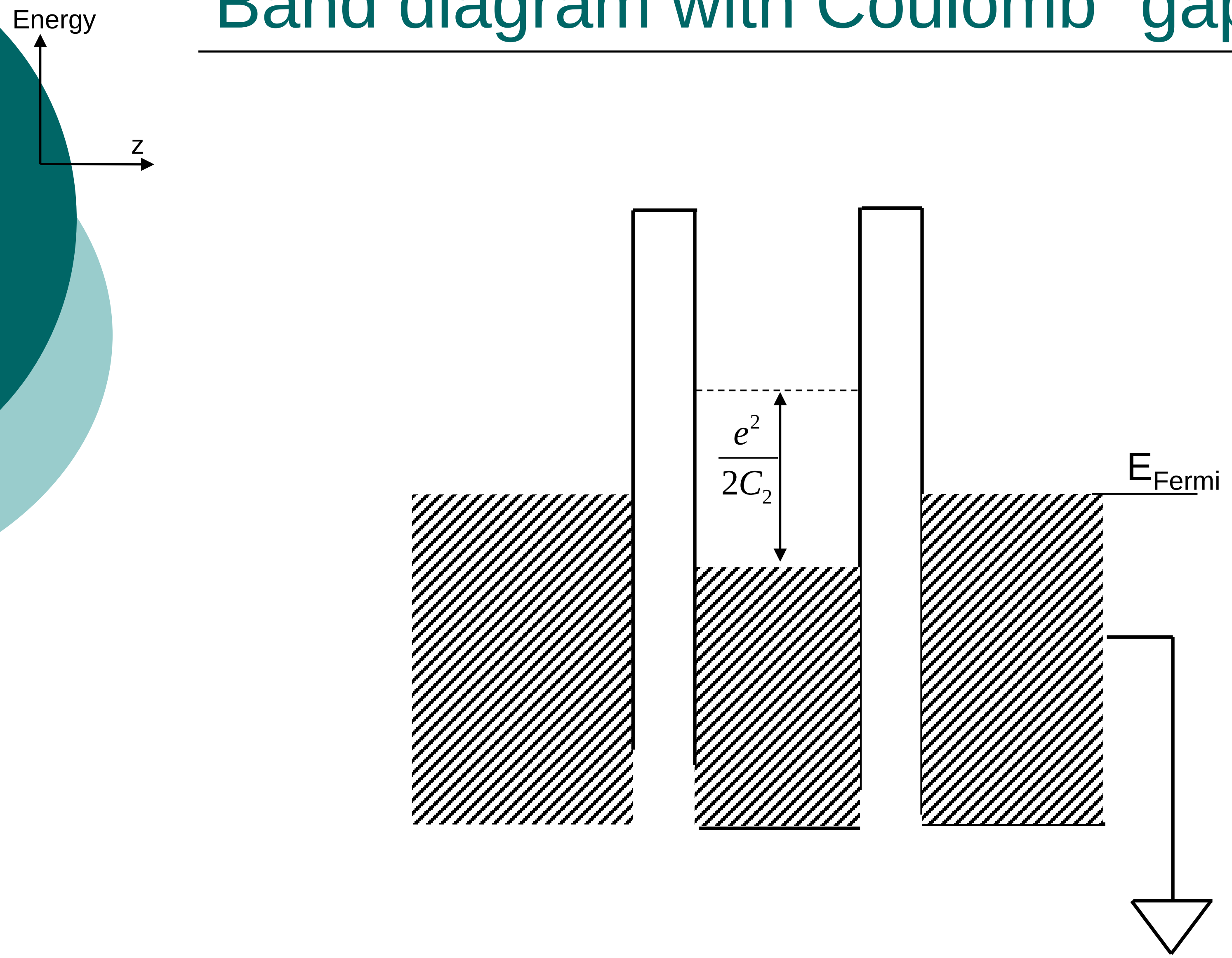
$$V = n\frac{e}{C_G} + \frac{e}{2C_G}$$



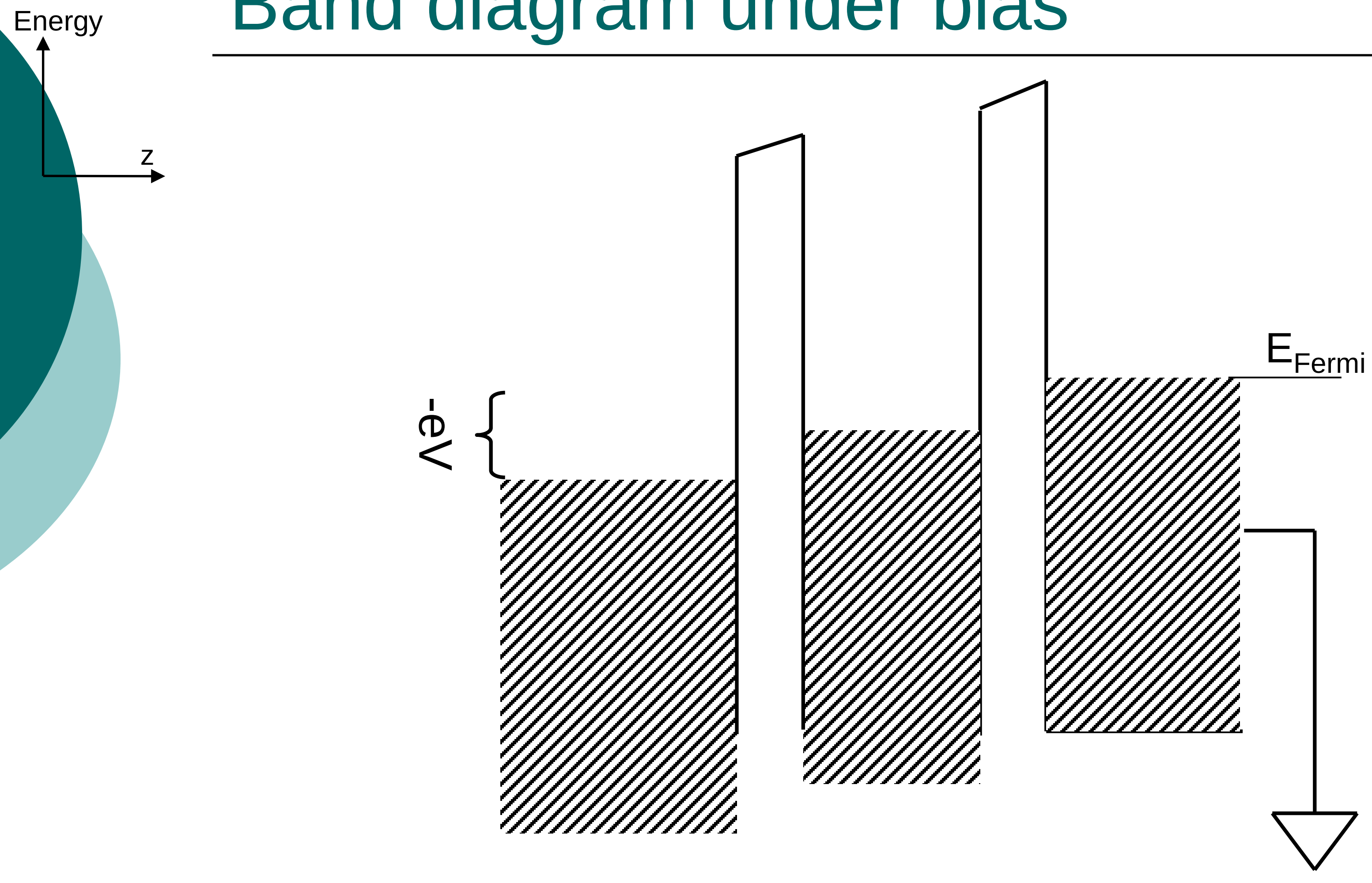
Band diagram



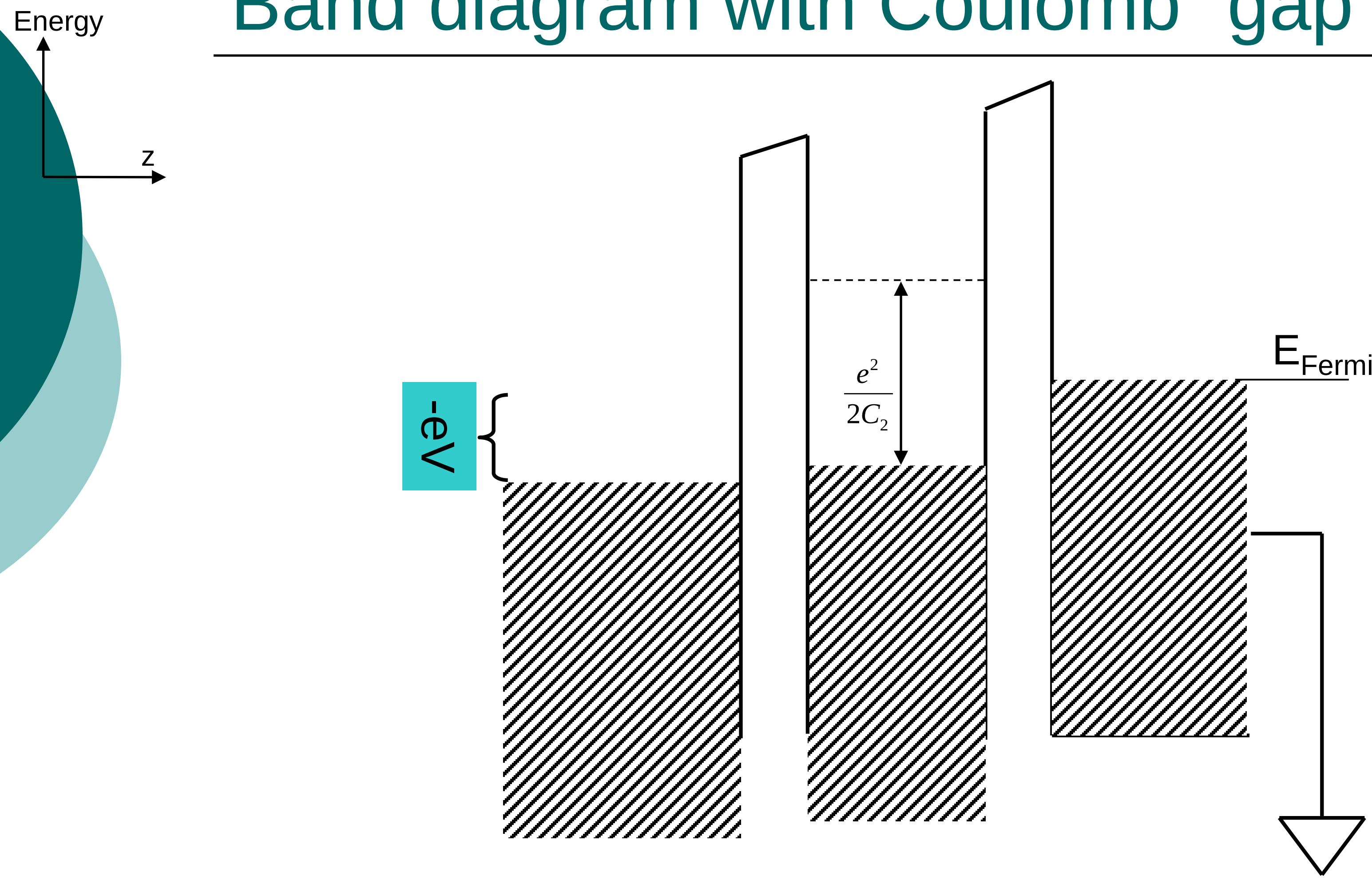
Band diagram with Coulomb "gap"



Band diagram under bias

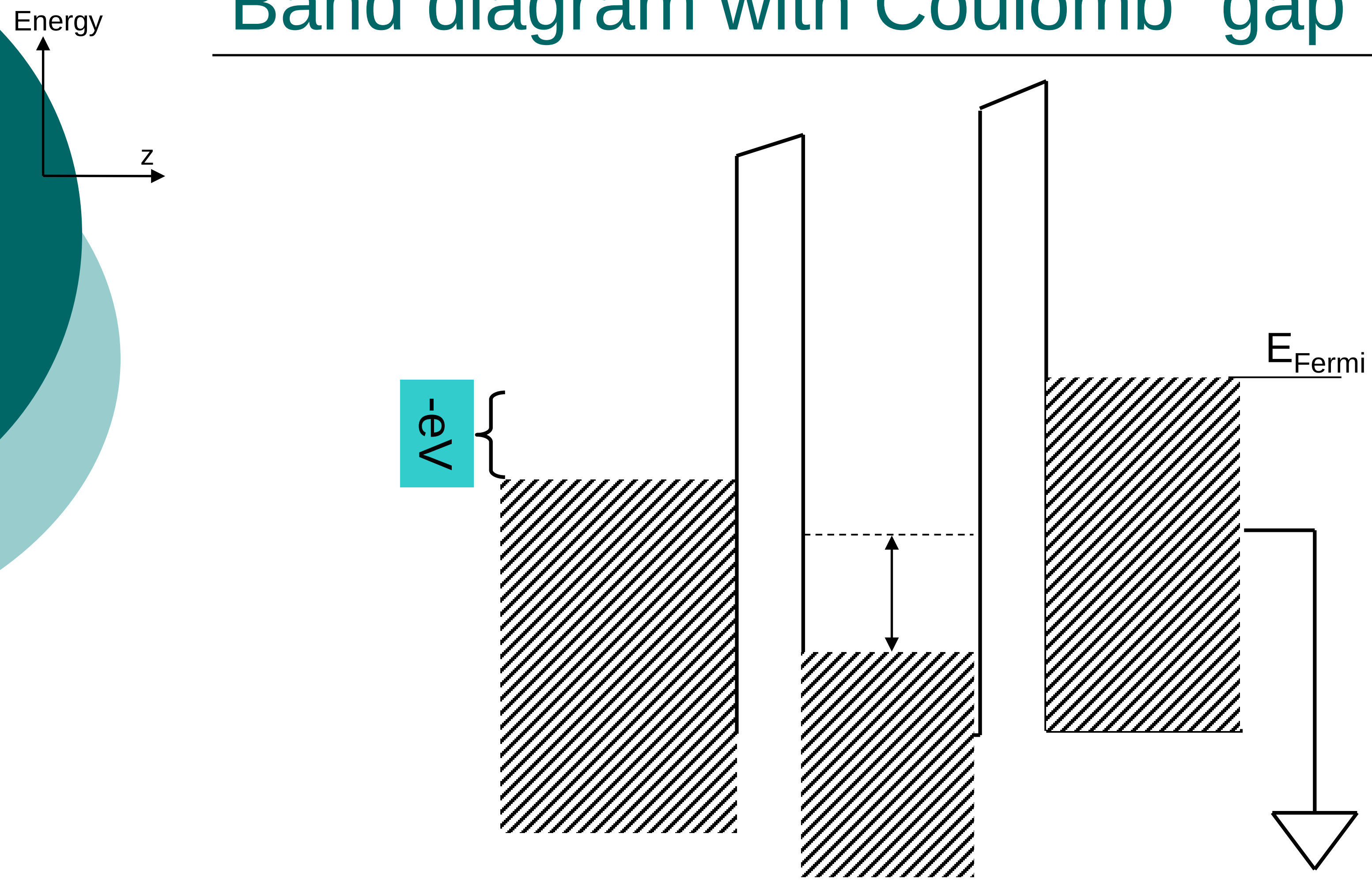


Band diagram with Coulomb "gap"



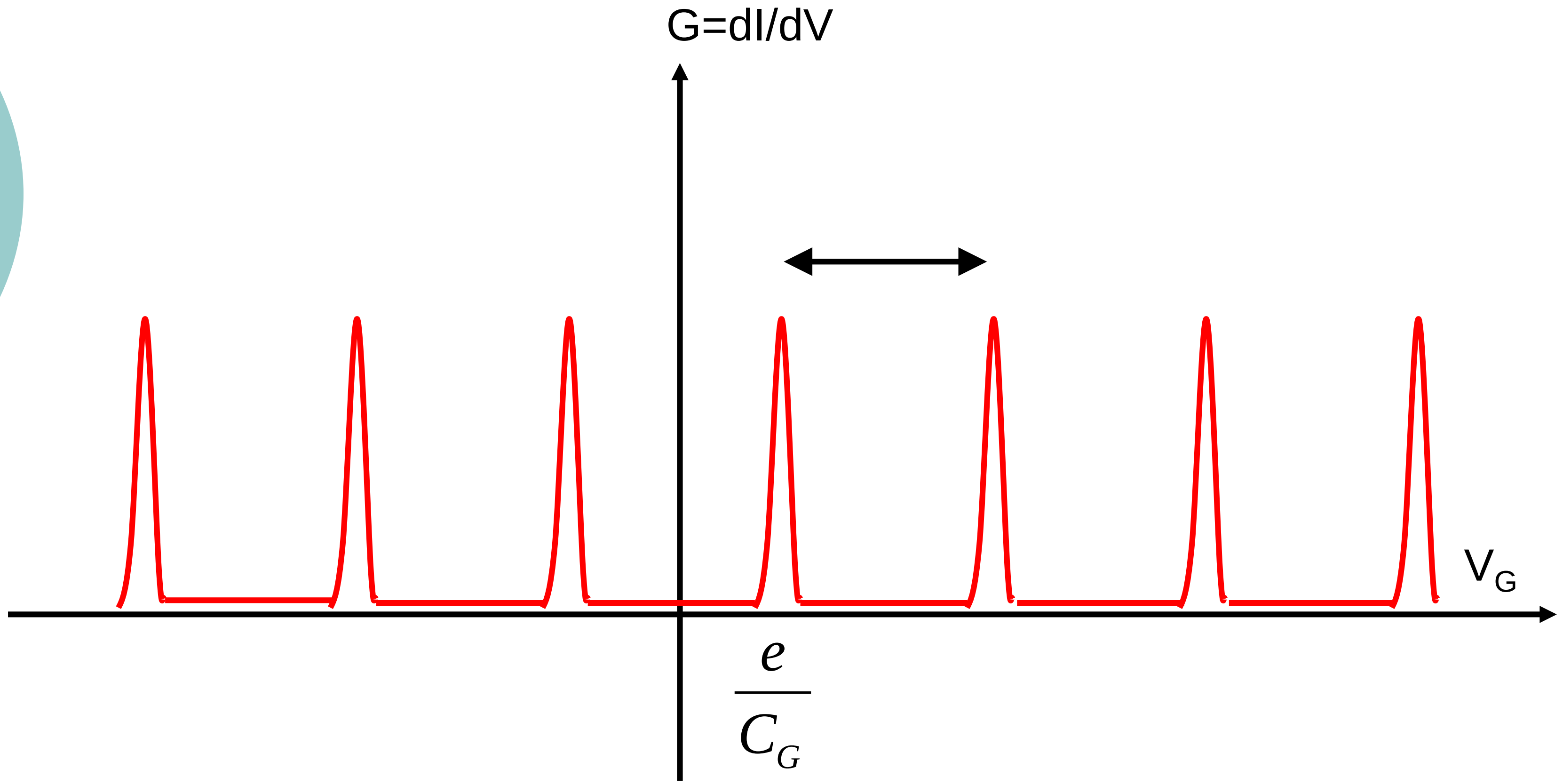
Gate voltage like a "plunger"
Moves island up/down by e^2/V_G

Band diagram with Coulomb "gap"



Gate voltage like a "plunger"
Moves island up/down by e^2/V_G

Zero bias conductance



Width of peaks set by temperature.

Coulomd diamond