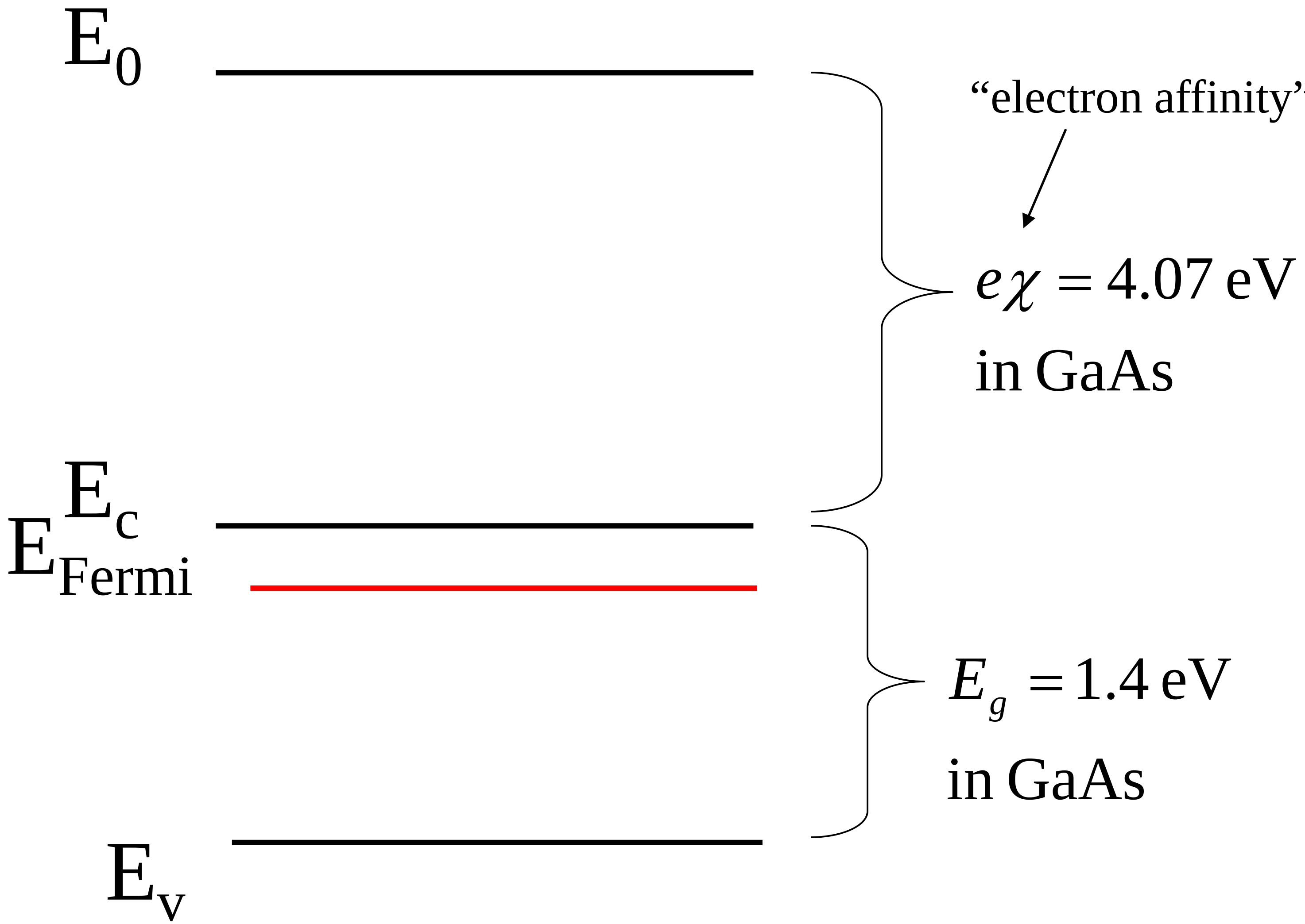
Lecture 9

2 dimensional electron gas (2DEG)

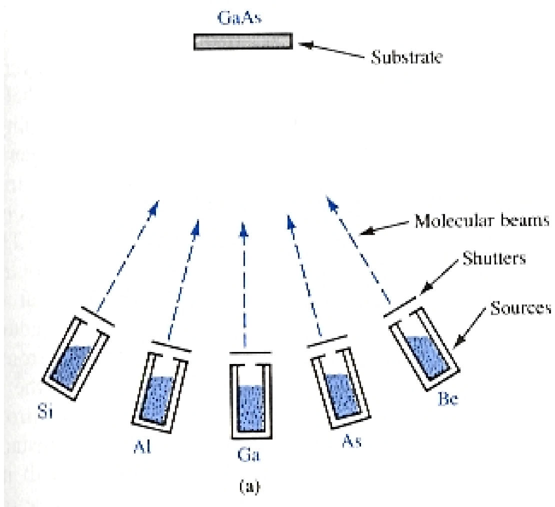
Readings this lecture covers

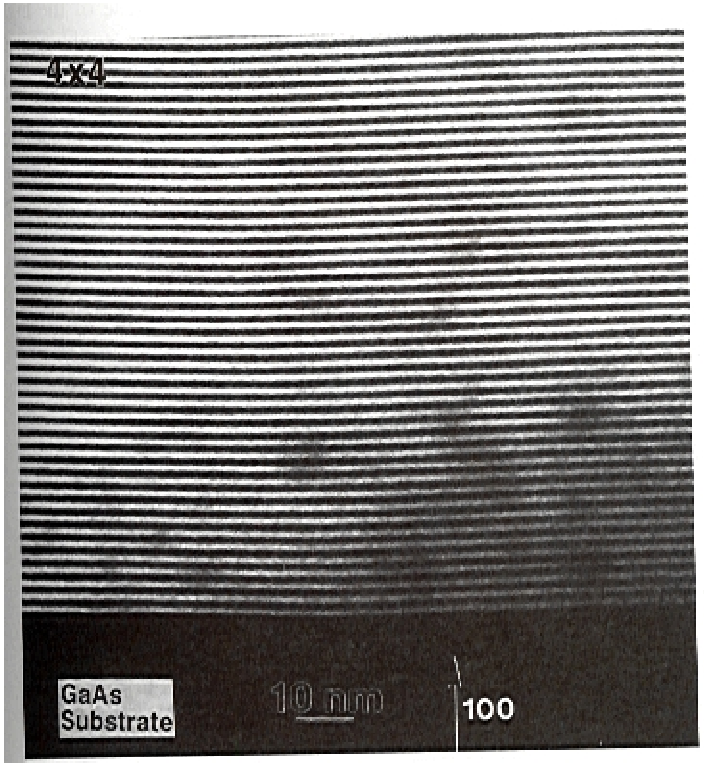
- Ferry, pp. 23-39
- Hanson, pp. 118-123

Vacuum level



MBE



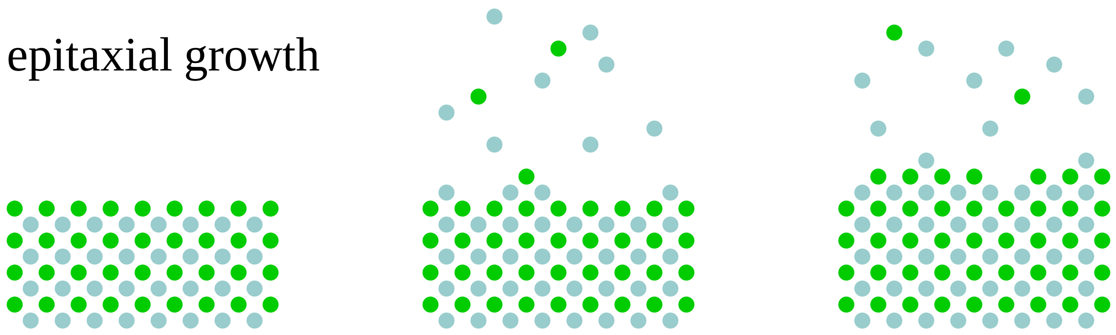


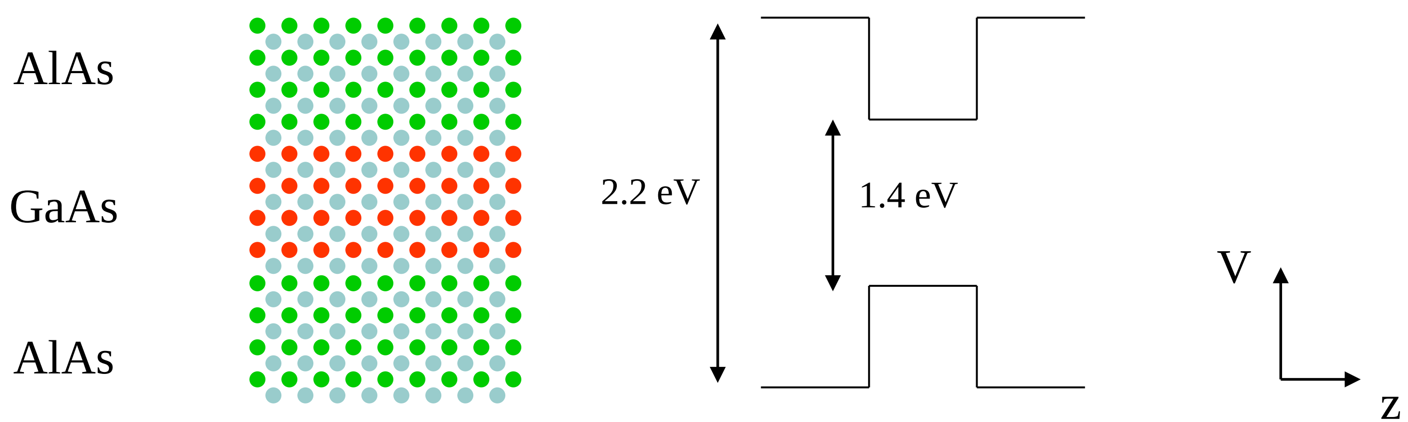
4 atom per layer!

(From Streetman, Solid State Electronic Devices)

MBE

epitaxial growth





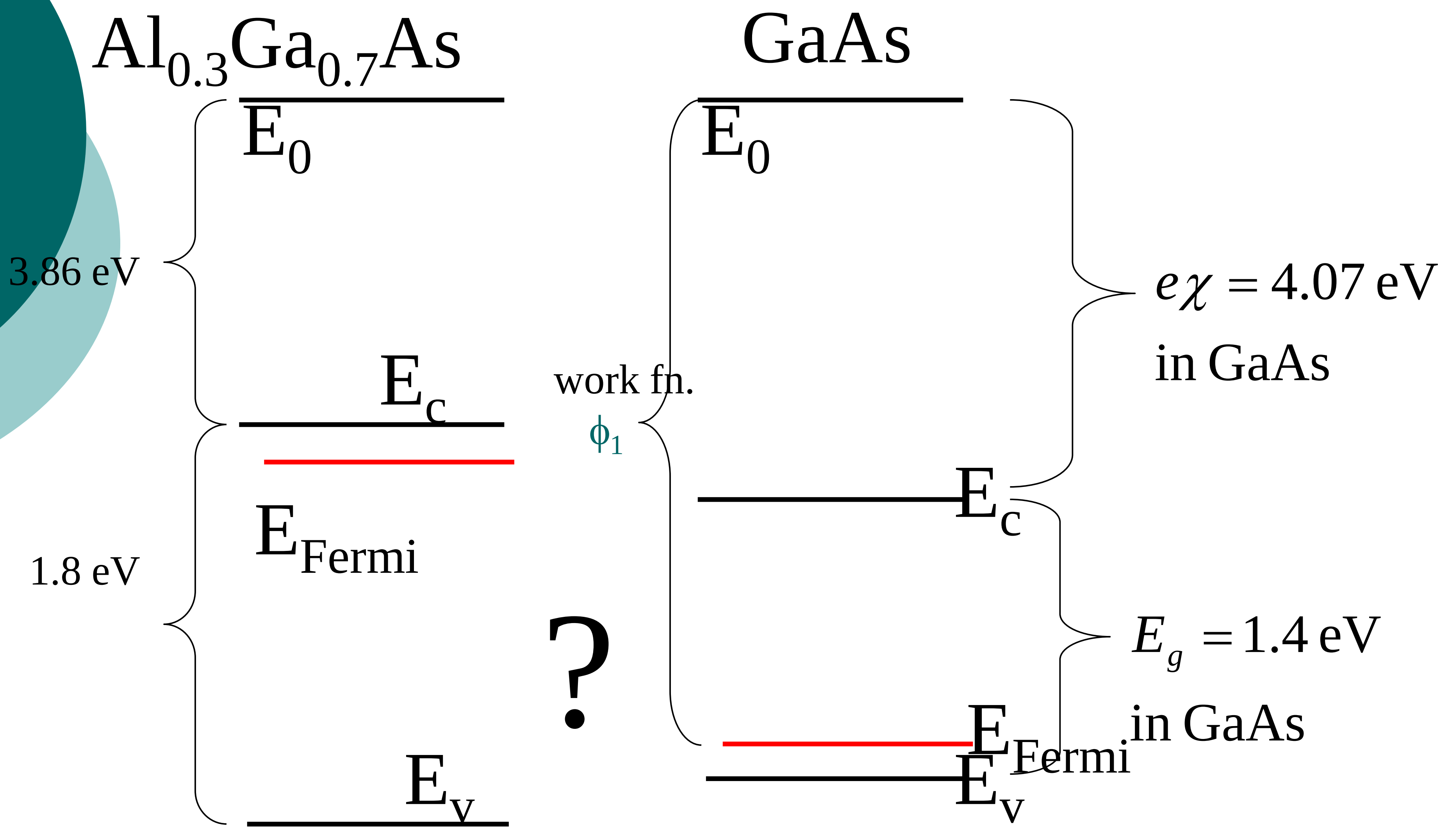
Also InP, InGaAs, InAlAs, InGaAsP …

Picture adapted from M. Lilly.

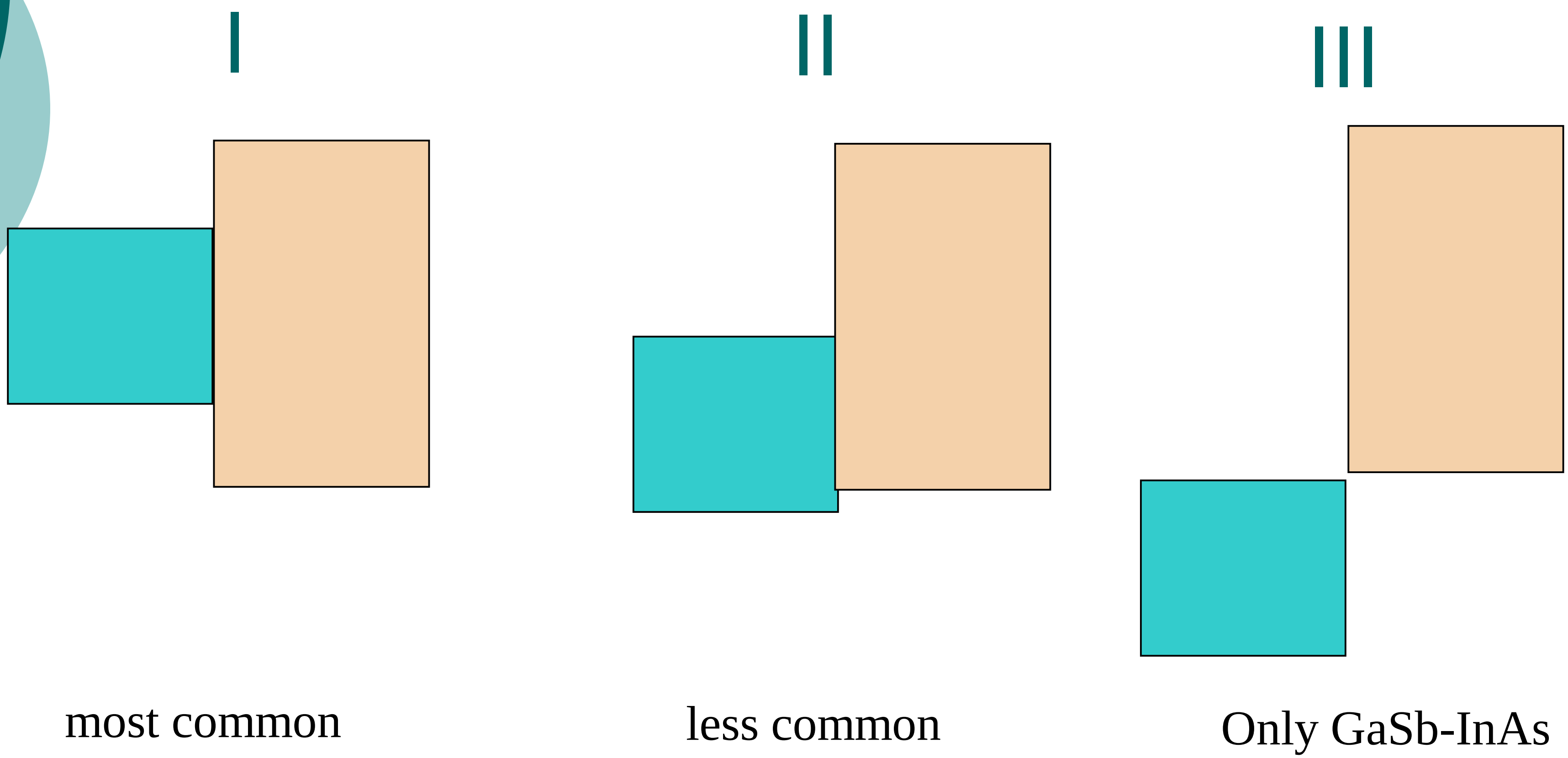
Heterojunction band diagrams

- Determine $\Delta E_c = \chi_1 - \chi_2$ (Vacuum levels line up)
- Determine $\Delta E_v = \Delta E_g - \Delta E_c$
- There will be some charge transfer and built-in electric field/voltage as in p-n homojunction
- Built in voltage $\phi = \phi_1 - \phi_2$
- Draw the diagram (You will in HW#2)

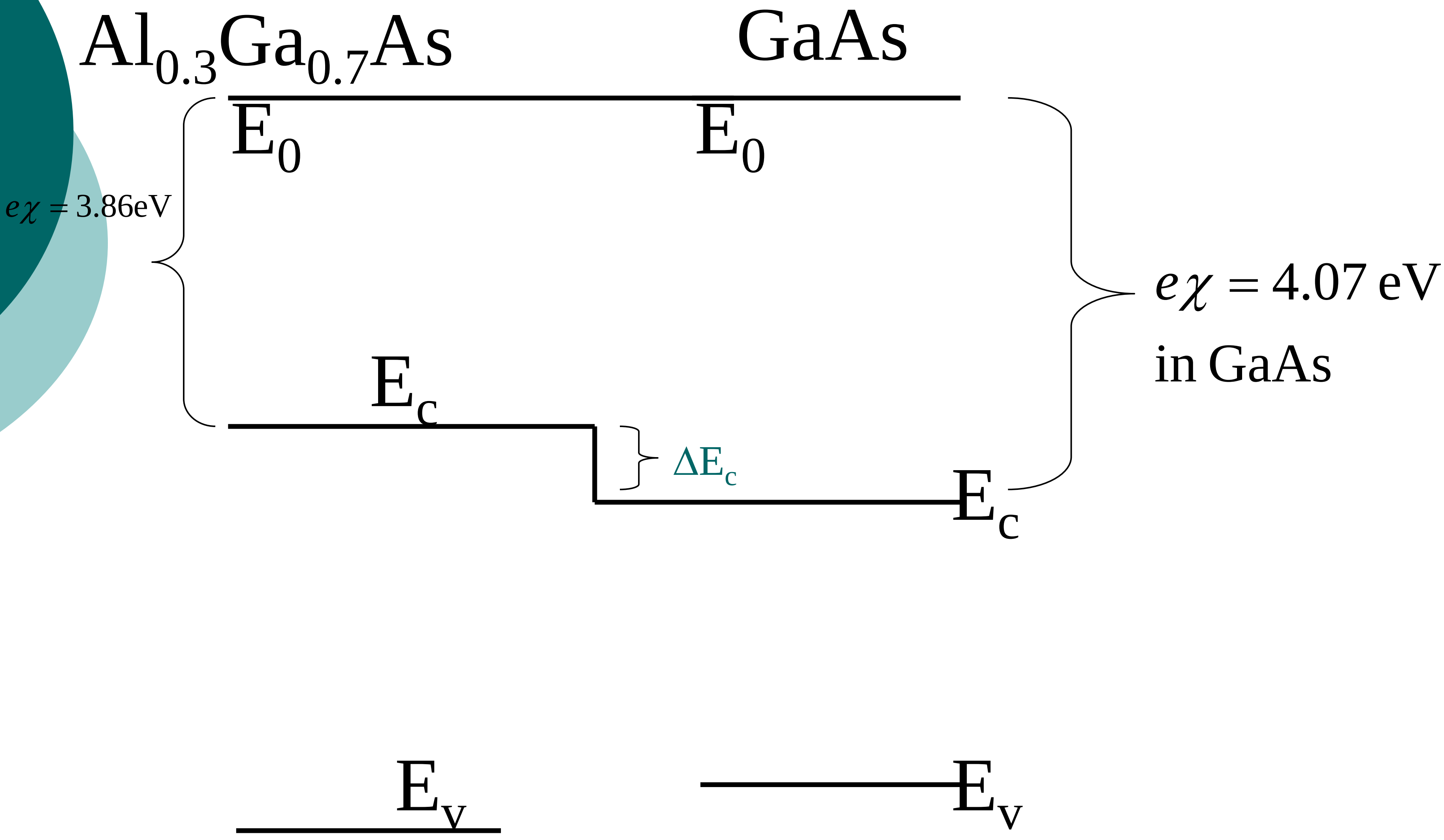
Intrinsic heterojunction



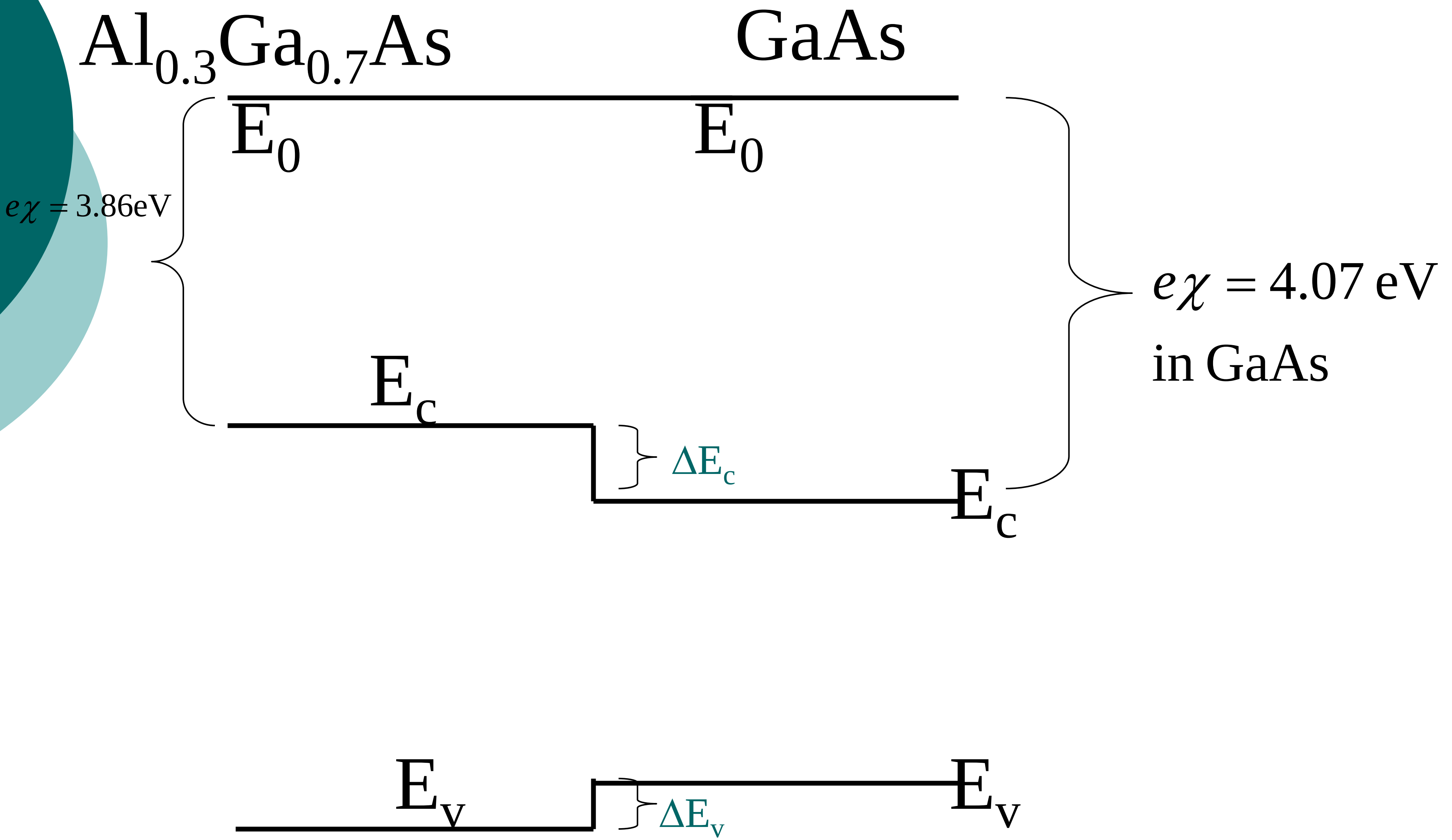
Types



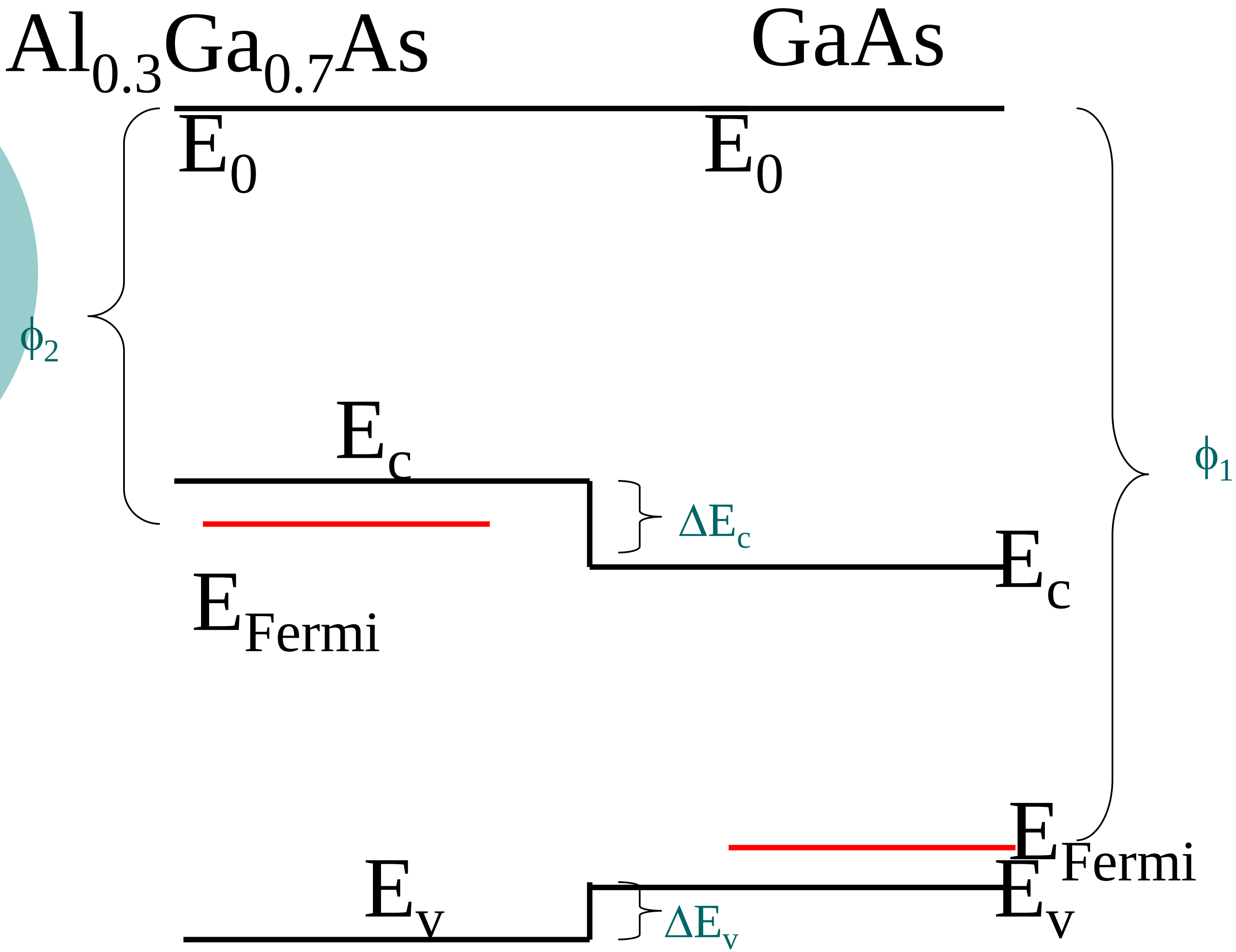
Determine $\Delta E_c = \chi_1 - \chi_2$



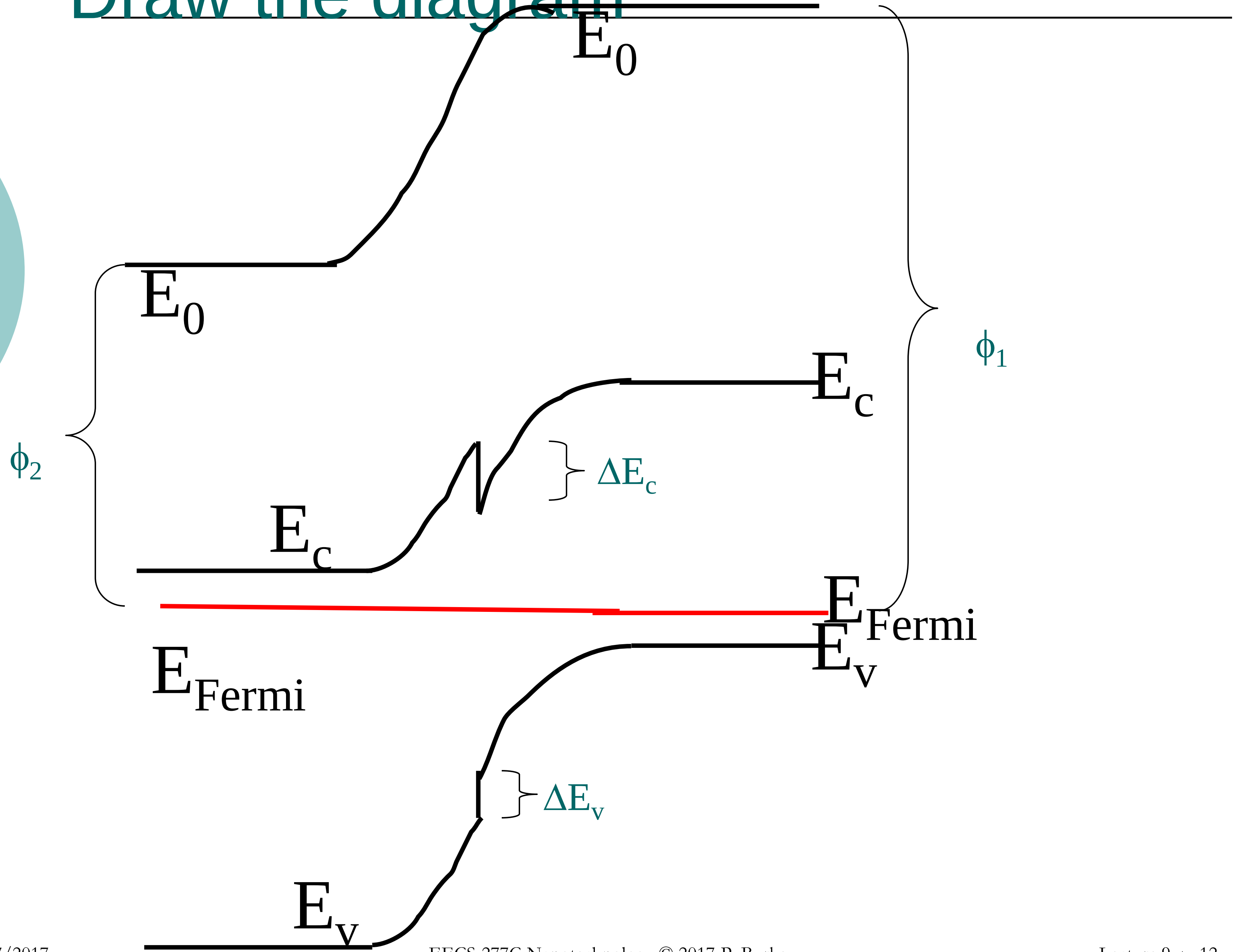
Determine $\Delta E_v = \Delta E_g - \Delta E_c$



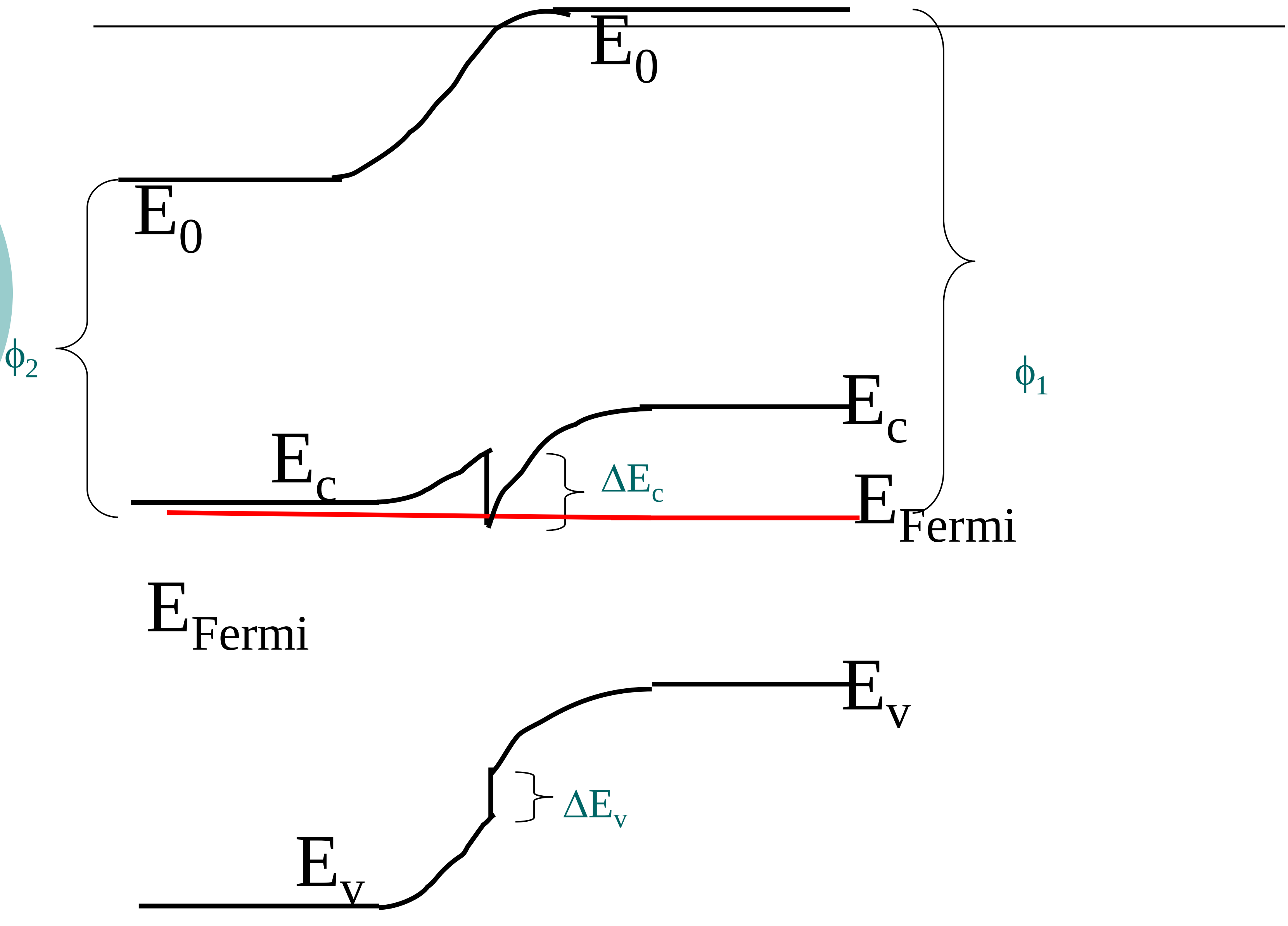
Built in voltage $\phi = \phi_1 - \phi_2$



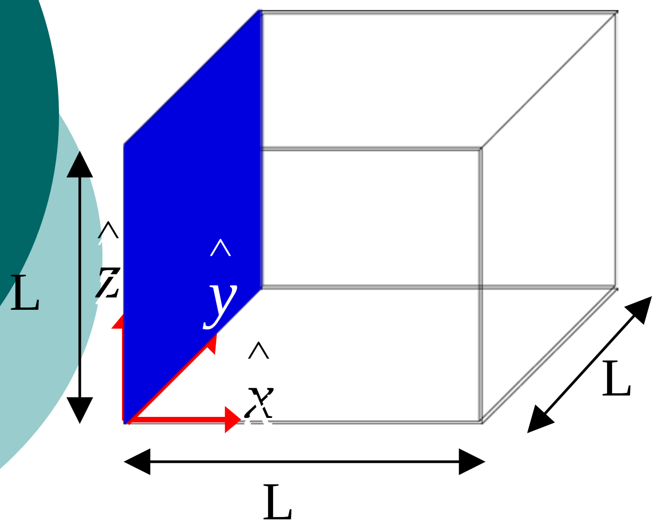
Draw the diagram



2DEG



Particle in a box:



$$\psi(\vec{r}) = (2i)^3 A \cdot \sin(k_{n_x} x) \cdot \sin(k_{n_y} y) \cdot \sin(k_{n_z} z)$$

$$k_{n_x} = n_x \pi / L$$

$$k_{n_y} = n_y \pi / L$$

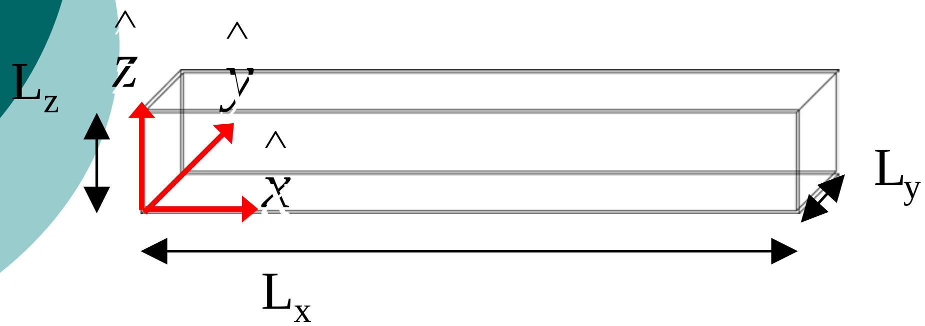
$$k_{n_z} = n_z \pi / L$$

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2 (\pi / L)^2}{2m} (n_x{}^2 + n_y{}^2 + n_z{}^2)$$

These are the allowed energy levels, or "quantum states"

Particle in a box

$$\psi(\vec{r}) = (2i)^3 A \cdot \sin(k_{n_x} x) \cdot \sin(k_{n_y} y) \cdot \sin(k_{n_z} z)$$



$$k_{n_x} = n_x \pi / L$$

$$k_{n_y} = n_y \pi / L$$

$$k_{n_z} = n_z \pi / L$$

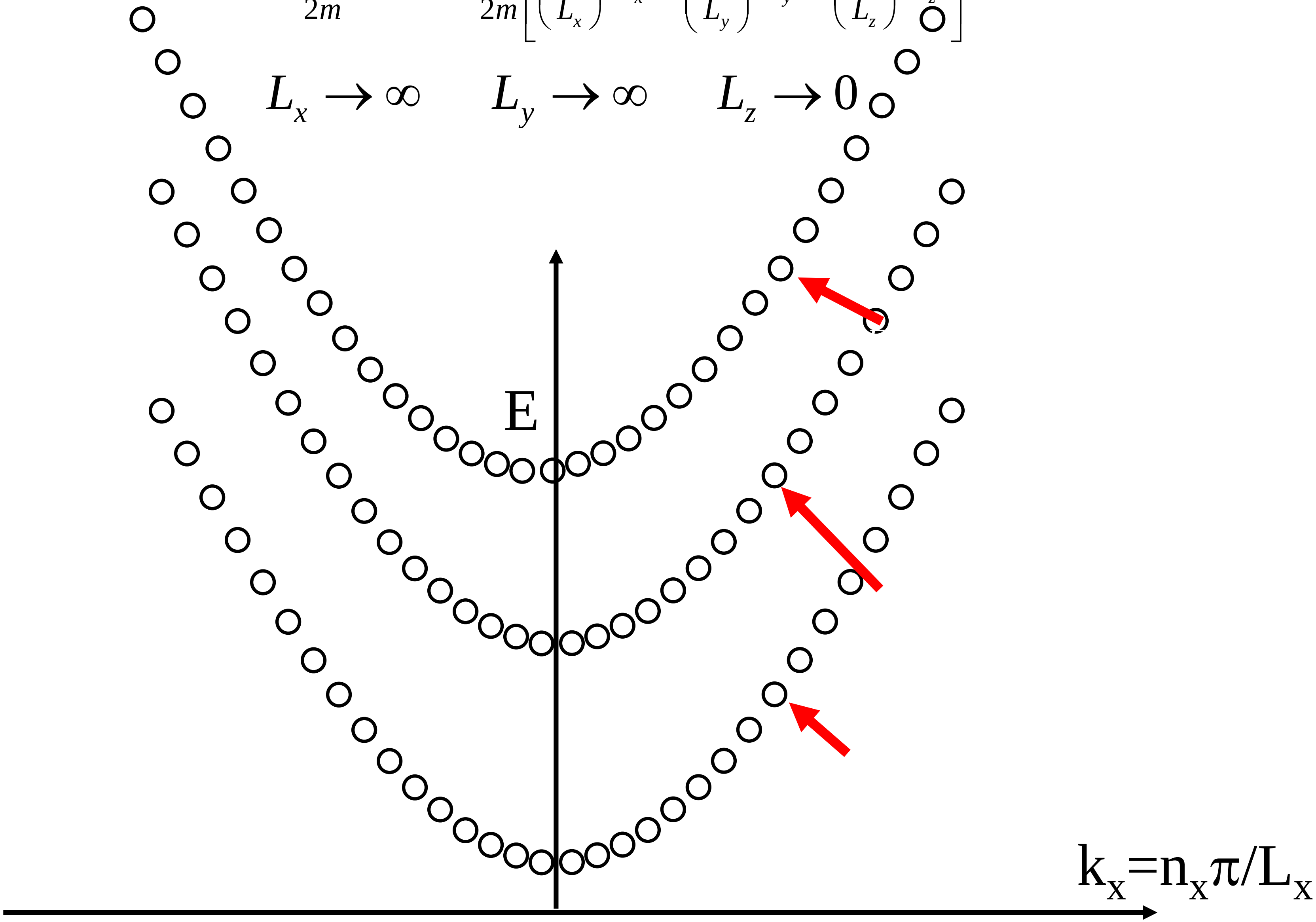
$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

These are the allowed energy levels, or “quantum states”

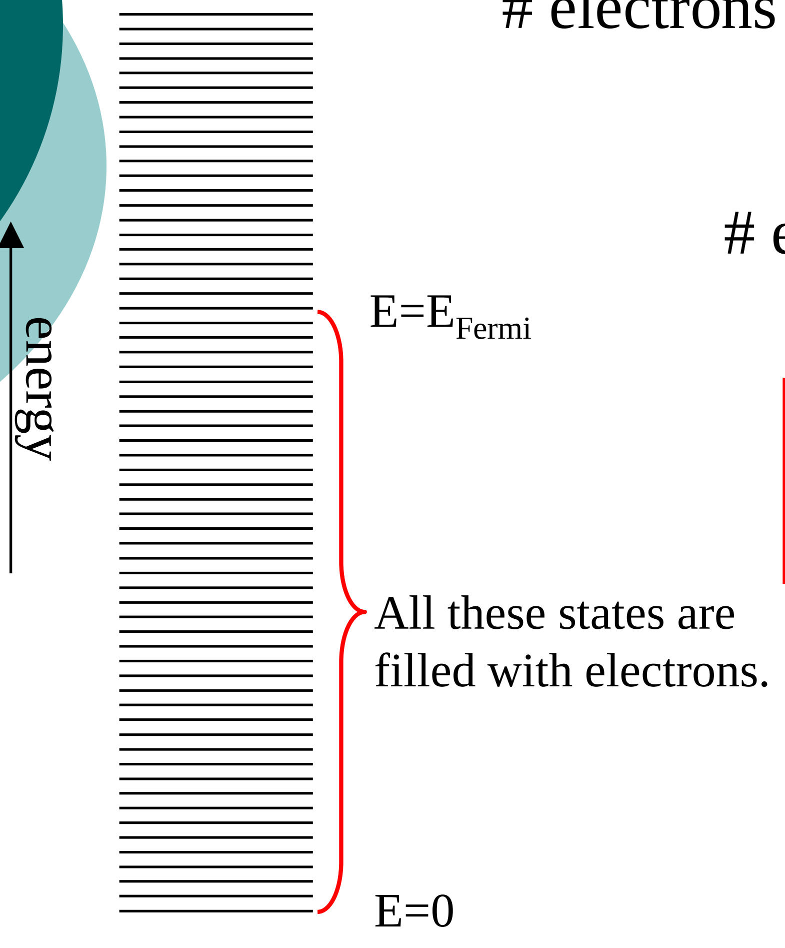
Limit:

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

$$L_x \to \infty \qquad L_y \to \infty \qquad L_z \to 0$$



Fermi energy in 3 dimensions



$$\# \text{ electrons} = \int_0^{E_f} N_E \, dE = \int_0^{E_f} L^3 \frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^{3/2}} \cdot E^{1/2} dE$$

$$\# \text{ electrons} = L^3 \frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^{3/2}} \frac{2}{3} E_f^{3/2}$$

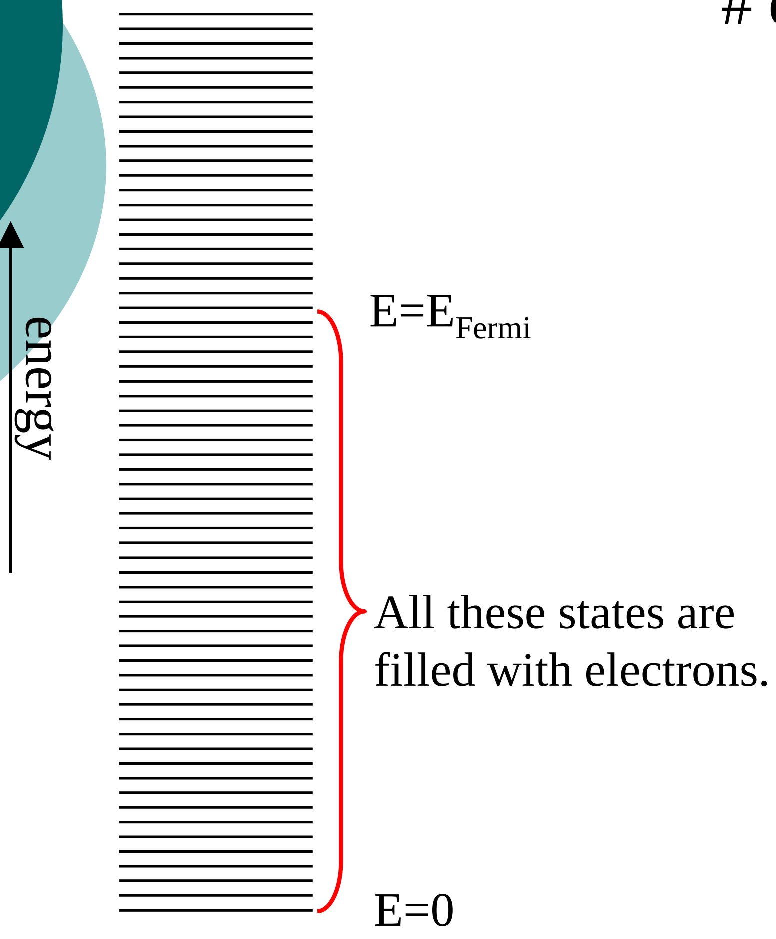
$$\Rightarrow E_f = \frac{\hbar^2 3^{2/3} \pi^{4/3}}{2m} \left(\frac{\# \text{ electrons}}{L^3} \right)^{2/3}$$

In a typical metal, L ~ 0.1 nm.

E_f ~ 10 eV

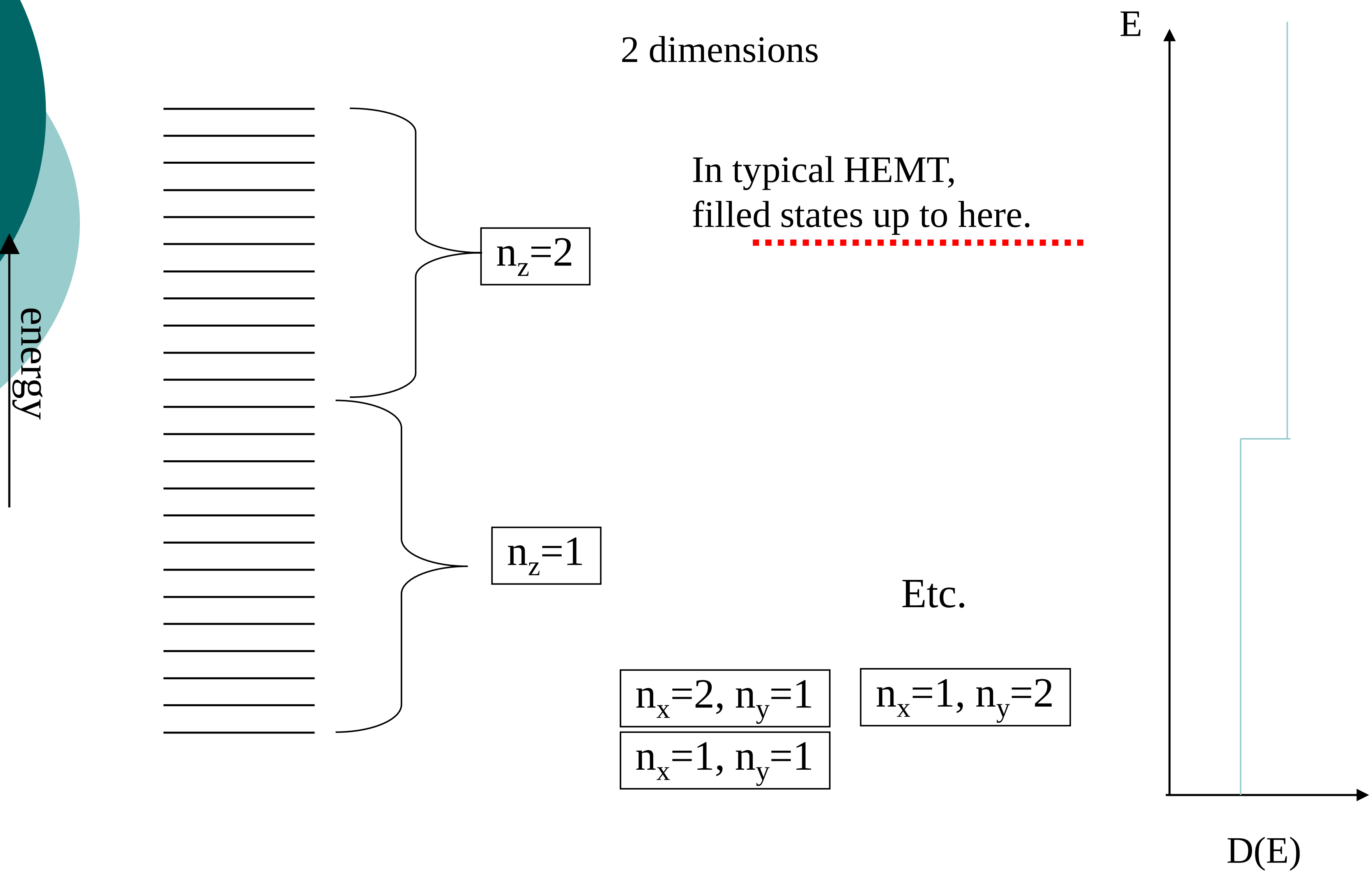
Fermi energy in 2 dimensions

$$\#\ \text{electrons} = \int_0^{E_f} N_{\mathrm{E}}\, \mathrm{f(E)dE} = ?$$

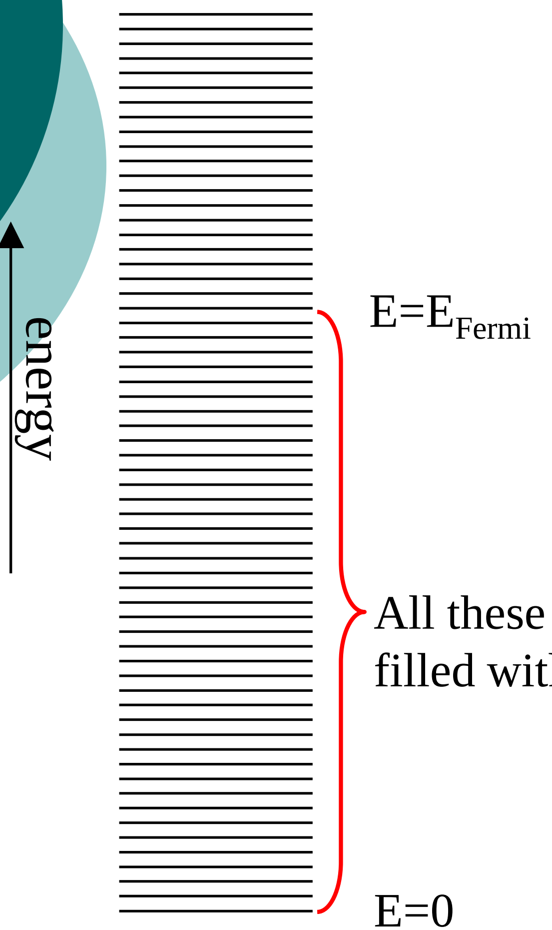


Need to evaluate integral numerically, just as in 3d.

Energy spectrum of free particles



Fermi energy in 2d



$$\#\ \text{electrons} = \int_0^{E_f} N_{\mathrm{E}}\, \mathrm{dE} = \int_0^{E_f} L^2 \frac{m}{\pi\hbar} dE$$

$$\#\ \text{electrons} = L^2 \frac{m}{\pi\hbar} E_f$$

$$\Rightarrow E_f = \frac{\hbar\pi}{m}\left(\frac{\#\ \text{electrons}}{L^2}\right)$$

In GaAs, 10^{11}cm^{-2} gives

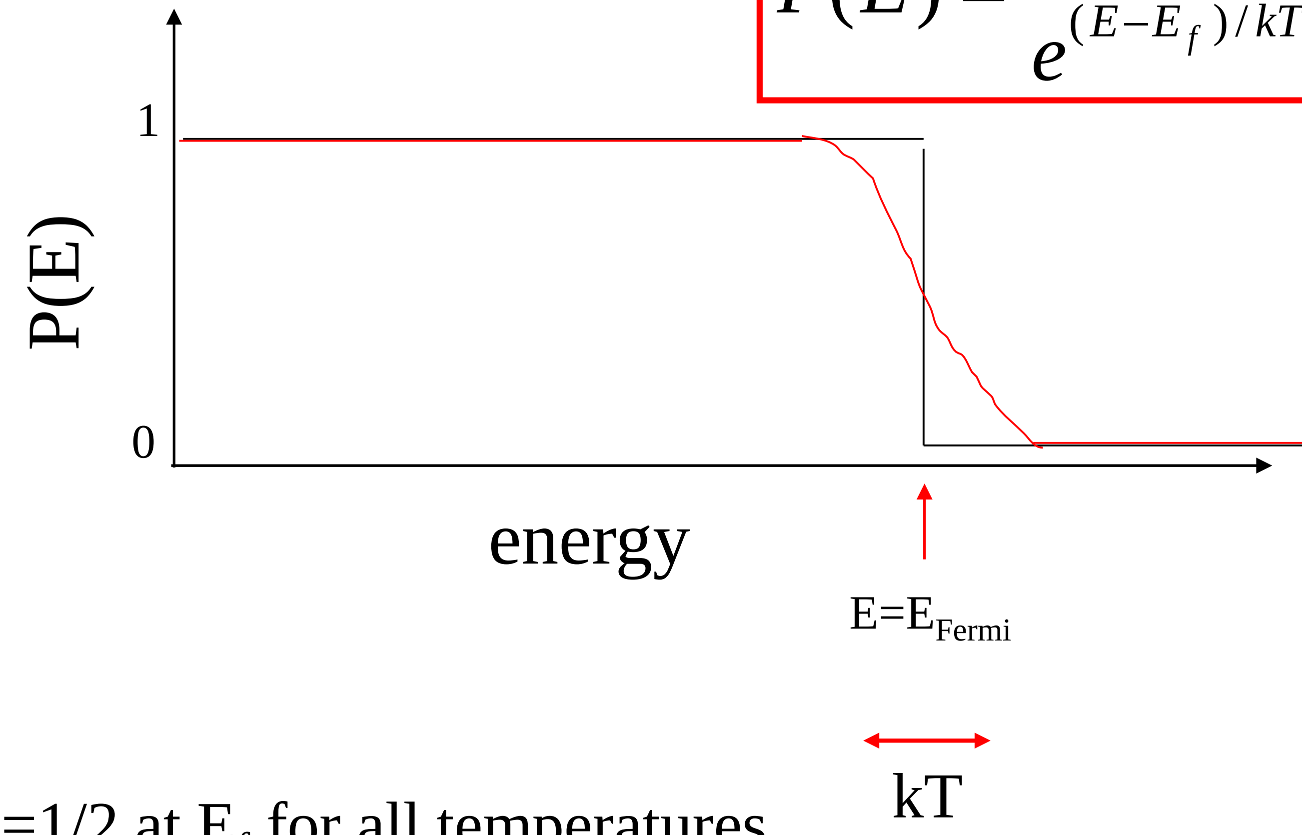
E_f ~ meV

But 10^{12}cm^{-2} gives more than first subband.

Discuss "subband", how above integral gets modified.

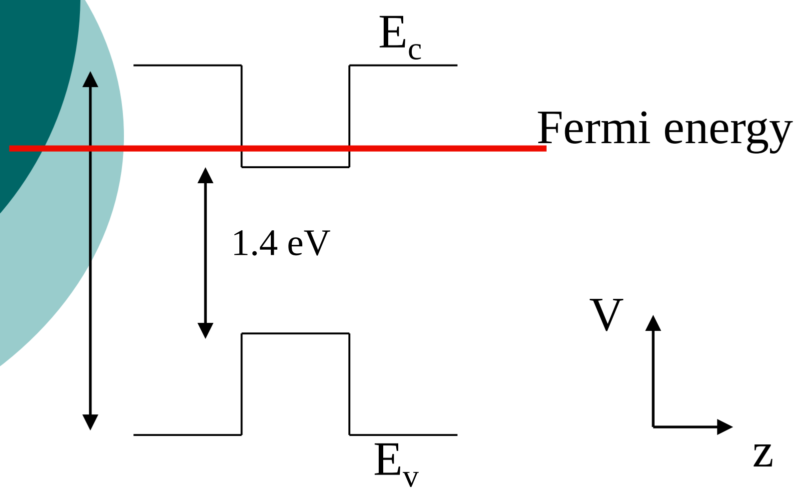
Fermi-Dirac

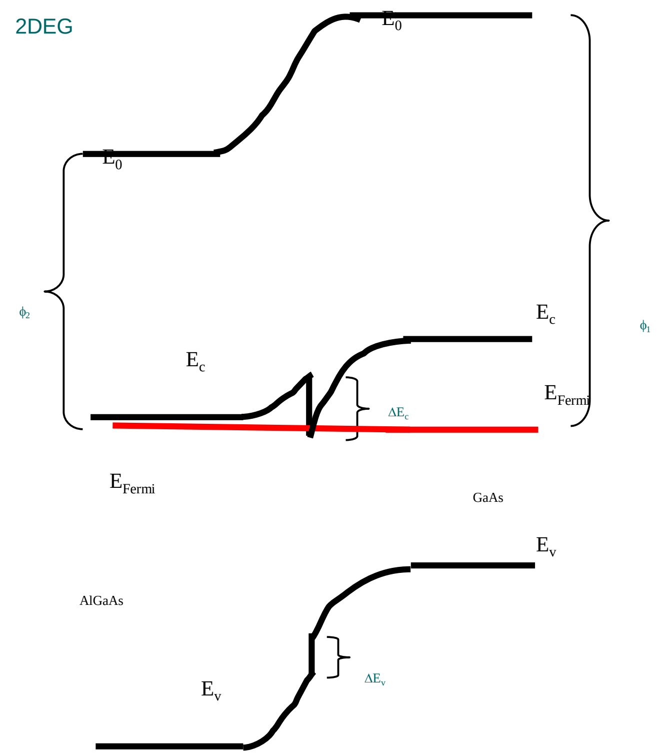
$$P(E)=\frac{1}{e^{(E-E_f)/kT}+1}$$



P=1/2 at E_f for all temperatures.

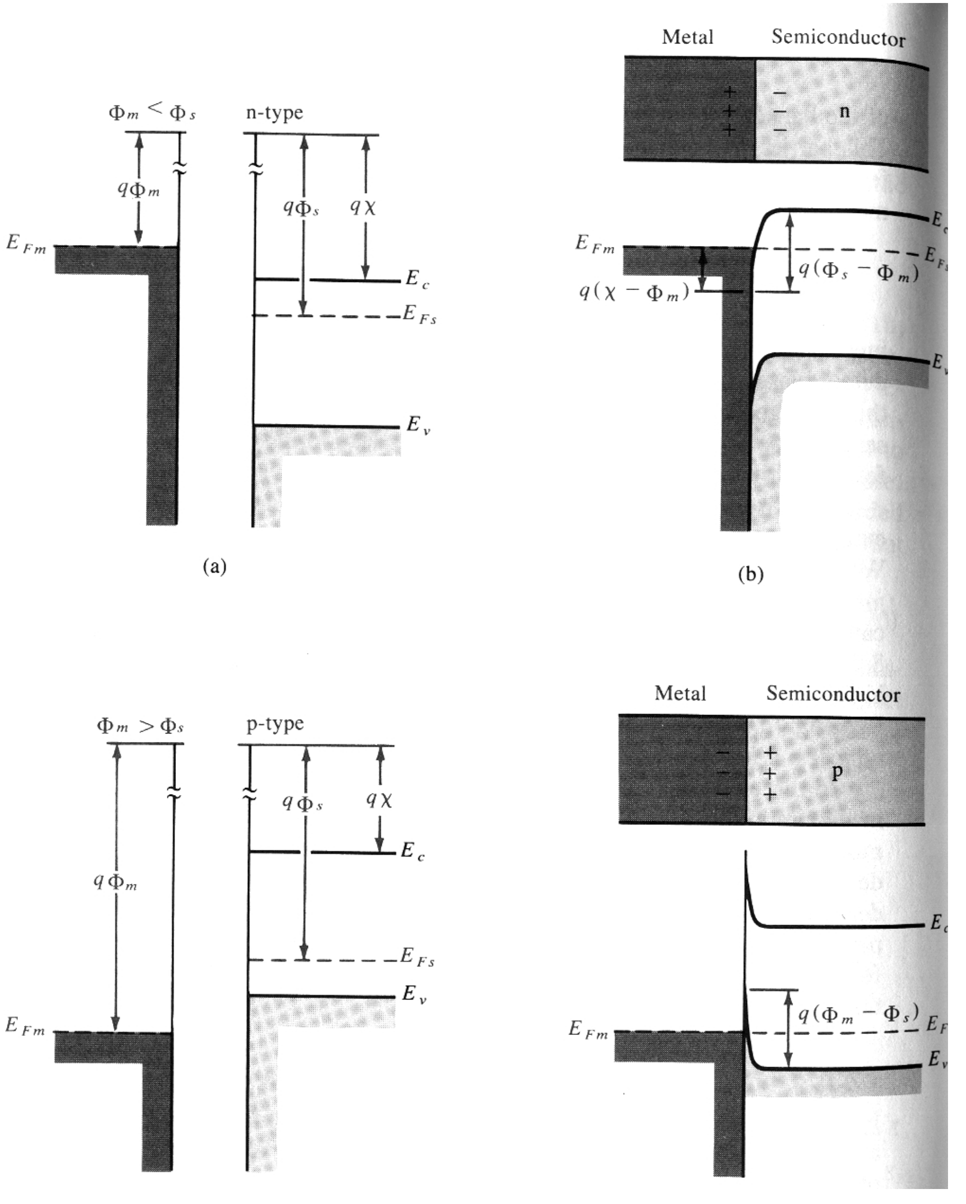
Triangle vs. square well:





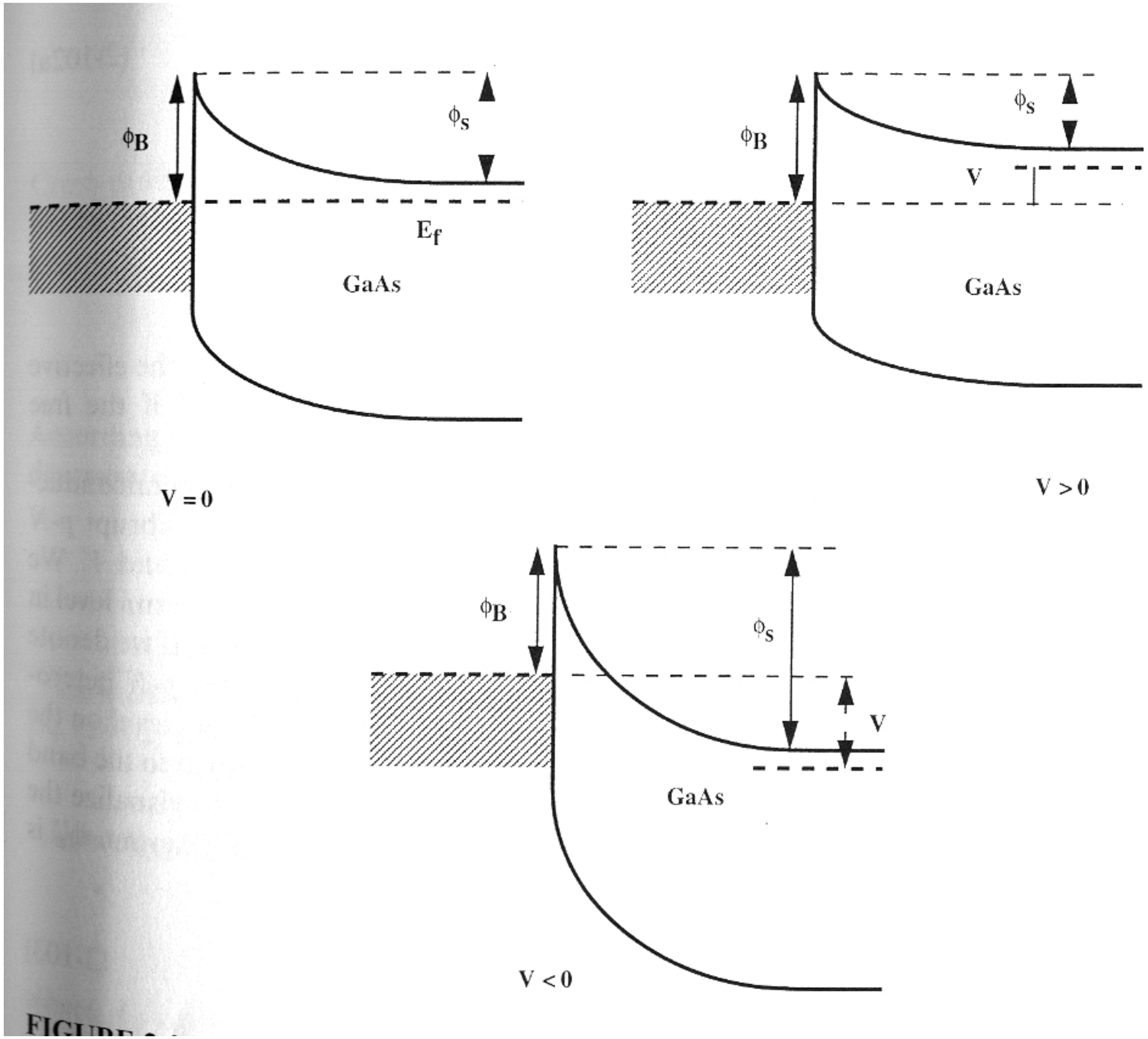
(Draw both bound states on board.
In particular discuss figure 5.21 from Liu.)
Also discuss shallow vs. wide wells on board.
(Typically 100 angstroms works.)
Discuss setback doping, mobility (time permitting).

Schottky barriers



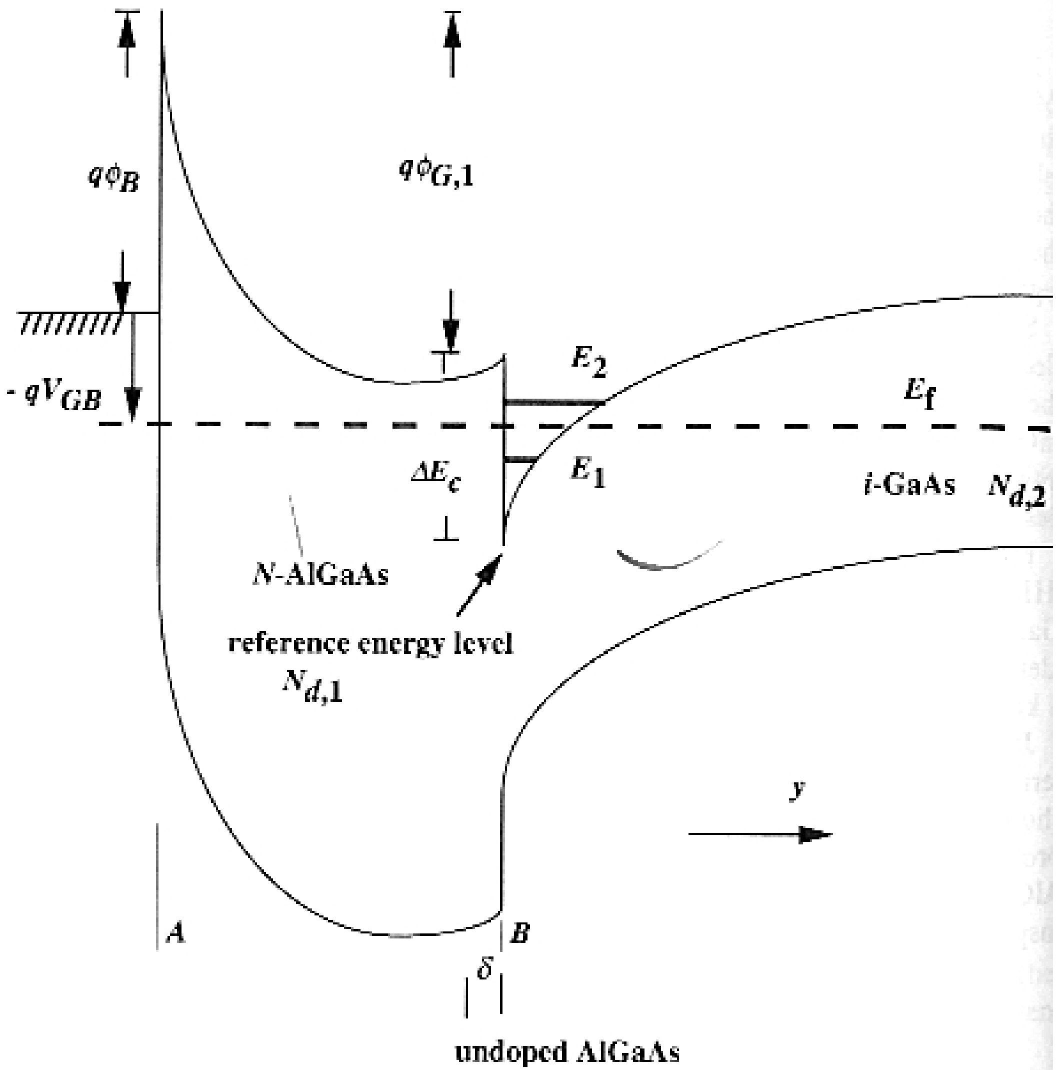
From Streetman

Schottky barriers



From Liu

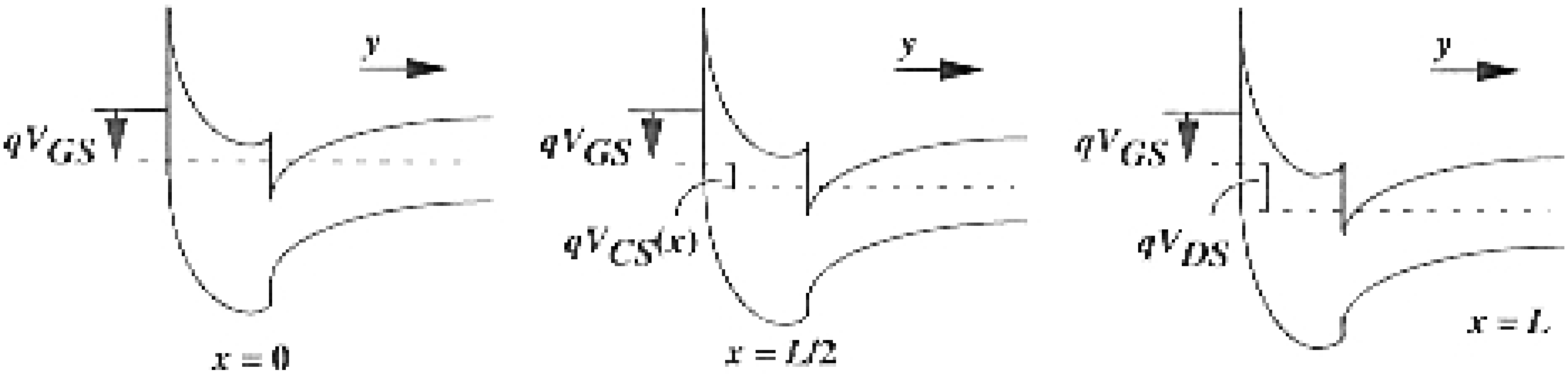
Band diagram



- Bias changes Fermi level
- hence density.
- Can "pinchoff"

From Liu.

Vary gate voltage



Changes Fermi energy which changes density.

(Draw better pictures on board.)

From Liu.

n_s vs E_f

After all that mumbo-jumbo, we know it is complicated.
We approximate it many times as:

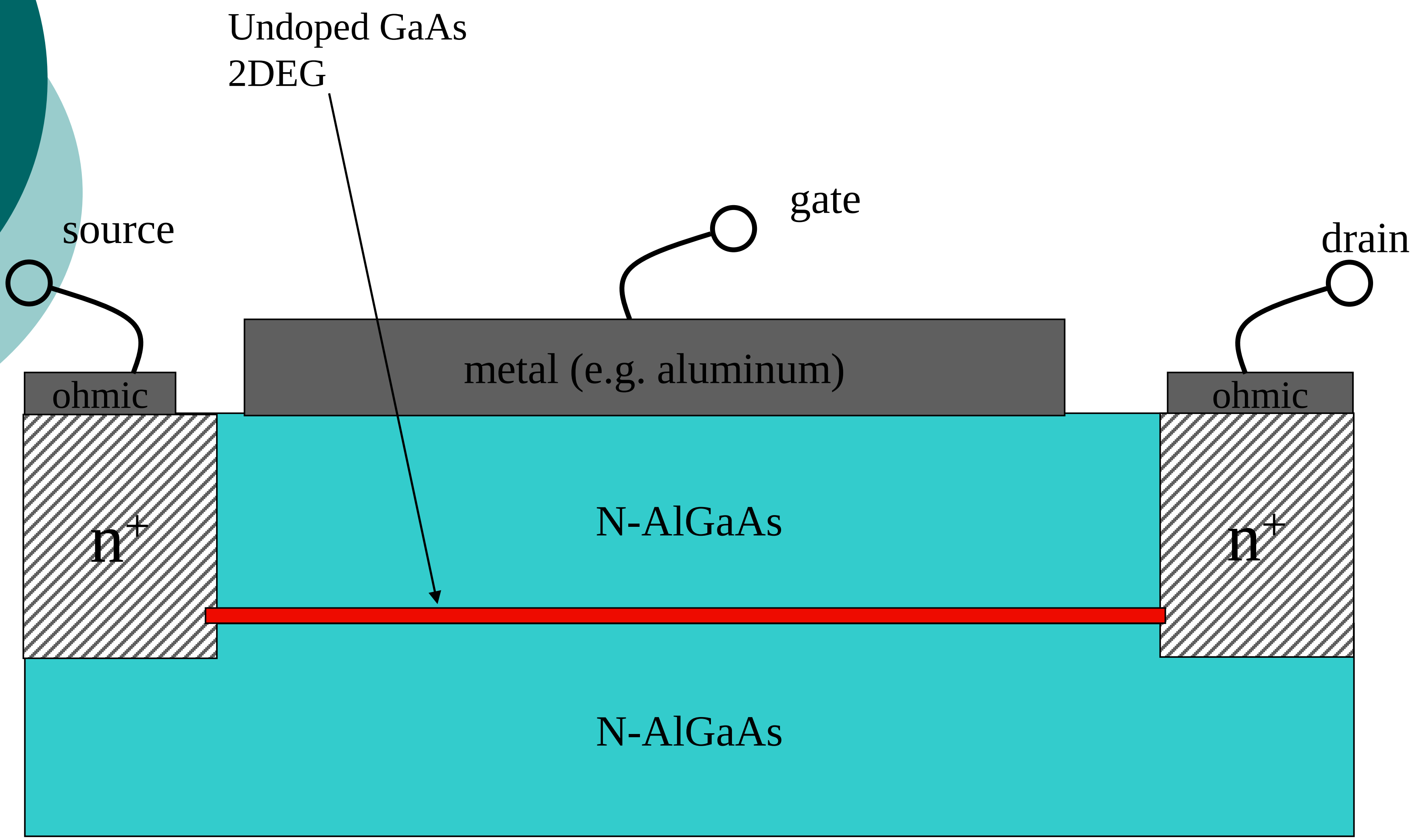
$$E_f(n_s) = E_{f,0} + a \cdot n_s$$

Density

$$en_s = \frac{\varepsilon}{t_b + \varepsilon a / e^2}\left(V_{GB} - V_T\right)$$

$$V_T \equiv \phi_B + \frac{E_{f,0}}{e} - \frac{eN_{d,1}}{2\varepsilon}(t_b - \delta)^2 - \frac{\Delta E_c}{e}$$

HEMT:


Undoped GaAs
2DEG
gate
source
drain
metal (e.g. aluminum)
ohmic
ohmic
N-AlGaAs
n+
n+
N-AlGaAs

Tunneling

- Resonant tunnel diodes
  - Draw band diagram, I-V on board
  - Fast (> 700 GHz)
- Optical/IR detectors
  - Like photoelectric effect
- Quantum cascade lasers
  - Levels within quantum wells lase