Q1	Q2	Q3	Q4	Q5	Q6	Total
/15	/30	/15	/15	/10	/15	/100

EECS / CSE 70A Final Exam

DO NOT BEGIN THE EXAM UNTIL YOU ARE TOLD TO DO SO.

Print your name on all pages.

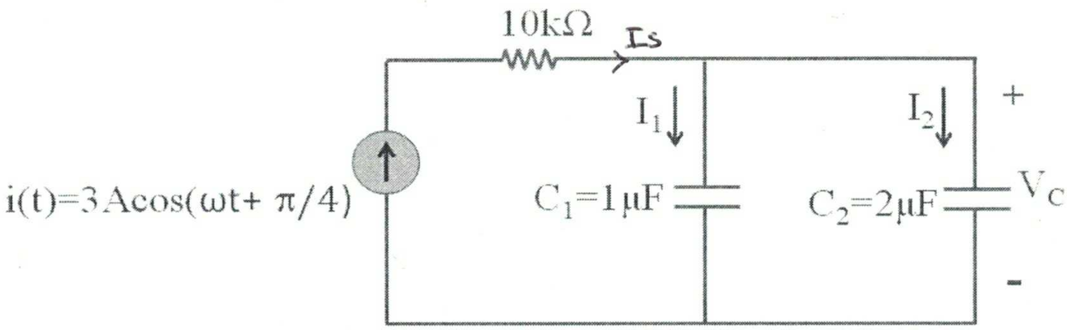
Write your solutions in clear steps with concise explanations.

radians :	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$	0
cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$	-1
tan	$\frac{\sqrt{0}}{\sqrt{4}}$	$\frac{\sqrt{1}}{\sqrt{3}}$	$\frac{\sqrt{2}}{\sqrt{2}}$	$\frac{\sqrt{3}}{\sqrt{1}}$	DNE	0

where $\sqrt{\cdot}$ always denotes the positive square root, and DNE means does not exist.

PROBLEM 1: (20 points)

Find phasor of $\mathbf{I_1}$, $\mathbf{I_2}$ and $\mathbf{V_C}$ and $V_C(t)$. Assume $\boldsymbol{\omega}$**=10^7 rad/s.**
(express your answers for phasor of $\mathbf{I_1}$, $\mathbf{I_2}$ and $\mathbf{V_C}$ in polar form $re^{j\theta}$)
(express all angles in **radians** not degrees)



$\mathbf{I_1}$	$1\,e^{j\pi/4}$ [A]
$\mathbf{I_2}$	$2\,e^{j\pi/4}$ [A]
$\mathbf{V_c}$	$0.1\,e^{-j\pi/4}$ [V]
$\mathbf{V_c(t)}$	$0.1\cos(10^7 t - \pi/4)$

$$I_s = 3e^{j\frac{\pi}{4}}$$

$$I_1 = I_s \cdot \frac{Z_{C_2}}{Z_{C_1}+Z_{C_2}} = I_s \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1}+\frac{1}{j\omega C_2}} = \boxed{I_s \frac{C_1}{C_1+C_2}} = \frac{1}{3} I_s$$ 1

$$\boxed{I_1 = 1e^{j\frac{\pi}{4}} \text{ [A]}}$$ 3

$$I_2 = I_s \frac{Z_{C_1}}{Z_{C_1}+Z_{C_2}} \rightarrow \boxed{I_2 = \frac{C_2}{C_1+C_2} I_s} \rightarrow \boxed{I_2 = 2e^{j\frac{\pi}{4}} \text{ [A]}}$$ 1 3

2

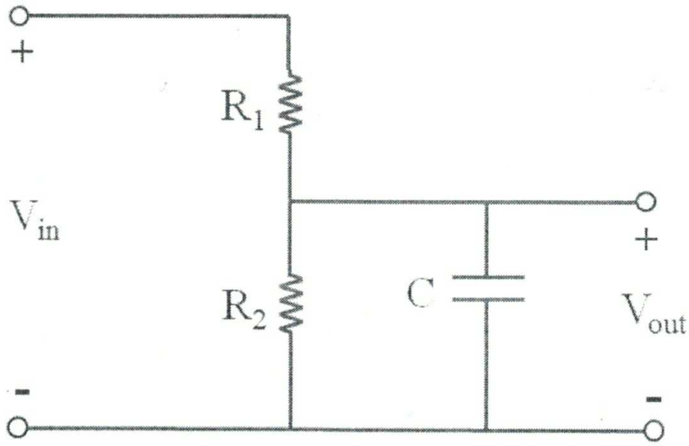
$$\boxed{V_C = I_1 \cdot Z_{C_1} \text{ or } I_2 Z_{C_2}} \rightarrow V_C = I_1 \cdot Z_{C_1} = 1e^{j\frac{\pi}{4}} \cdot \frac{1}{j\,10^7\,10^{-6}} = -j\,0.1e^{j\frac{\pi}{4}} \rightarrow$$

$$\boxed{V_C = 0.1\,e^{-j\frac{\pi}{4}} \text{ [V]}}$$ 3

$$\boxed{V_C(t) = 0.1\cos(10^7 t - \frac{\pi}{4})}$$ 2

PROBLEM 2: (30 points)

In the Circuit below the transfer function is defined as $\mathbf{H}(\boldsymbol{\omega}) = \frac{V_{out}}{V_{in}}$. Find the following.



a) Find $\mathbf{H}(\boldsymbol{\omega})$ in terms of R1, R2 and C.

$$\frac{1}{jwC} \| R_2 = \frac{R_2 \cdot \frac{1}{jwC}}{R_2 + \frac{1}{jwC}} = \frac{R_2}{1+jwCR_2}$$

$$H(w) = \frac{\frac{1}{jwC} \| R_2}{\left(\frac{1}{jwC} \| R_2\right) + R_1} = \frac{\frac{R_2}{1+jwCR_2}}{\frac{R_2}{1+jwCR_2} + R_1}$$

4

$$H(w) = \frac{R_2}{R_2 + R_1(1+jwCR_2)} = \boxed{\frac{R_2}{R_2 + R_1 + jwCR_1R_2}}$$

or

or $\boxed{H(w) = \frac{1}{\left(1+\frac{R_1}{R_2}\right) + jwCR_1}}$

6

H(ω)	$\frac{R_2}{(R_1+R_2)+jwCR_1R_2}$

b) Find $|\mathbf{H}(\omega)|$ in terms of R_1, R_2 and C.

$$H(w) = \boxed{\frac{R_2}{\sqrt{(R_1+R_2)^2 + (wCR_1R_2)^2}}}$$ 5

or

$$\boxed{\frac{1}{\sqrt{\left(1+\frac{R_1}{R_2}\right)^2 + (wCR_1)^2}}}$$

$\lvert\mathbf{H}(\omega)\rvert$	$= \frac{R_2}{\sqrt{(R_1+R_2)^2 + (wCR_1R_2)^2}}$

c) Find $\angle\mathbf{H}(\omega)$ in terms of R_1, R_2 and C.

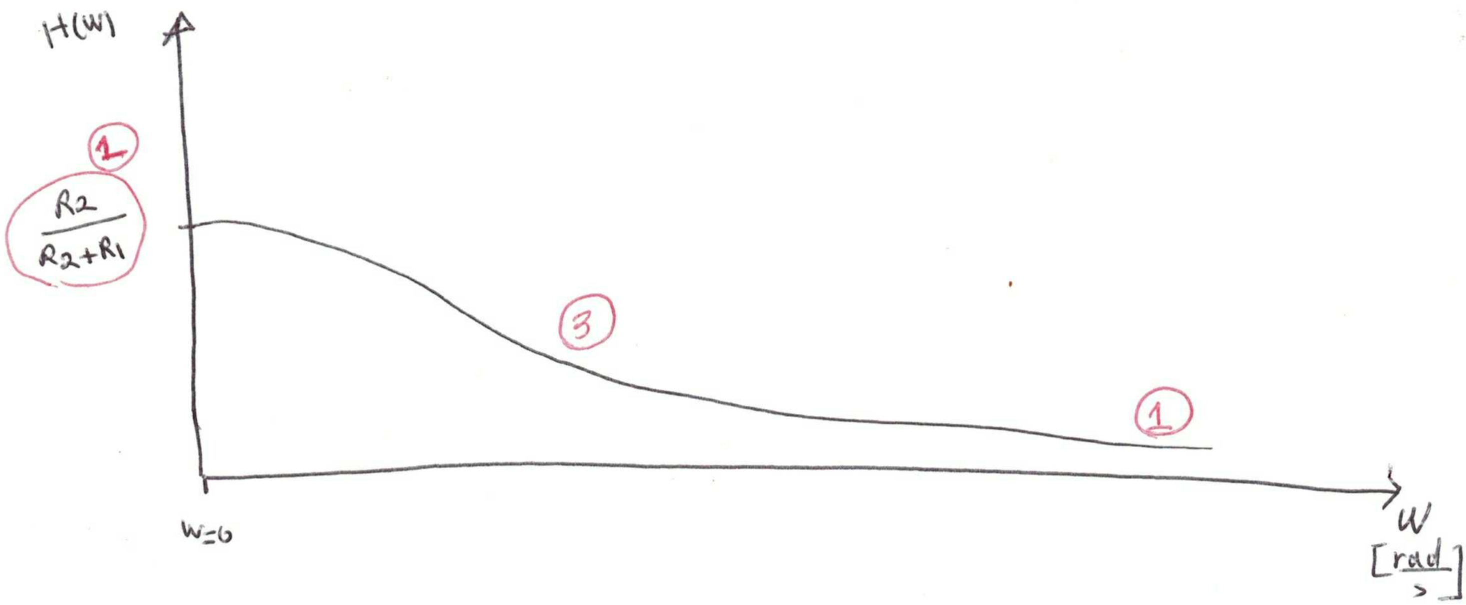
$$\boxed{\angle H(w) = -Arctg^{-1}\left(\frac{wCR_1R_2}{R_1+R_2}\right)}$$ 5

or

$$\boxed{\angle H(W) = -Arctg^{-1}\left(\frac{wCR_1\cancel{R_2}}{1+\frac{R_1}{R_2}}\right)}$$

$\angle\mathbf{H}(\omega)$	$-Arctg^{-1}\left(\frac{wCR_1R_2}{R_1+R_2}\right)$

d) Plot $|\mathbf{H}(\boldsymbol{\omega})|$ on lin-lin plot. Mark important points on the axes.



e) What is the value of $|\mathbf{H}(\boldsymbol{\omega})|$ at $\boldsymbol{\omega} = \mathbf{0}$ and at $\boldsymbol{\omega} \to \infty$ in terms of R_1, R_2 and C.

$$H(0) = \frac{R_2}{R_2+R_1}$$

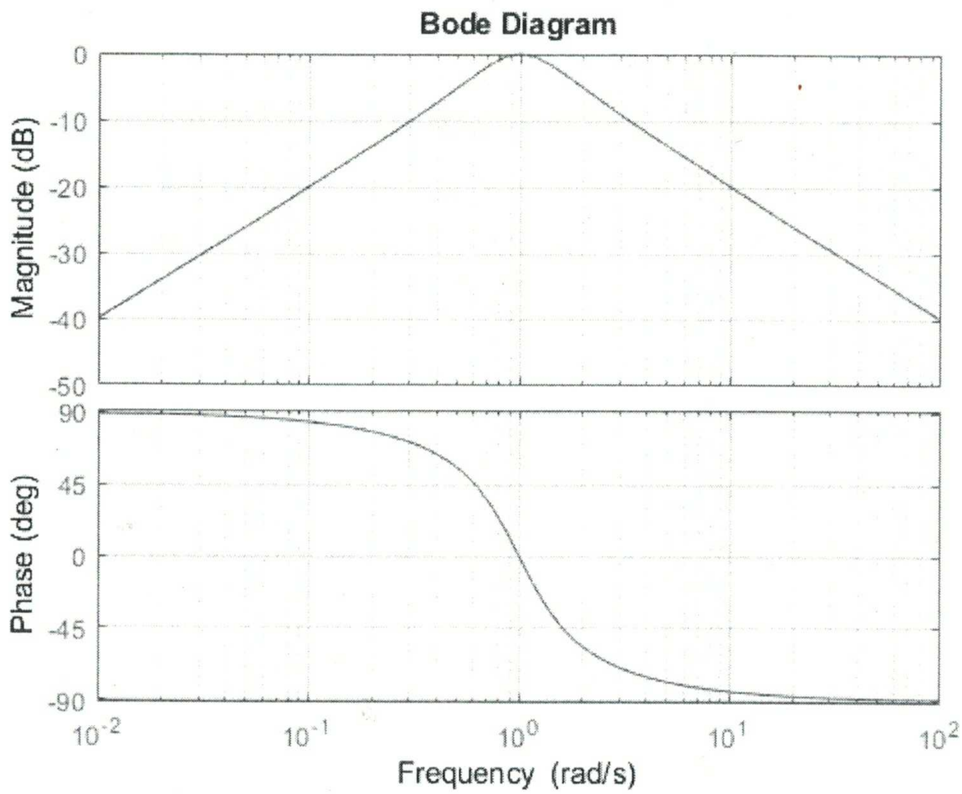
$$\lim_{W \to \infty} H(W) = 0$$

$\lvert\mathbf{H}(\mathbf{0})\rvert$	$\frac{R_2}{R_1+R_2}$	2.5
$\lim\limits_{\omega\to\infty} \lvert\mathbf{H}(\boldsymbol{\omega})\rvert$	0	2.5

PROBLEM 3: (15 points)

The Bode plots of the transfer function is given below, find $V_{out}(t)$ for the following $V_{in}(t)$.

$$\mathbf{V_{in}(t) = 1\,mV\cos(10^{-2}\frac{rad}{s}t + \frac{\pi}{4}) + 10\,mV\cos(10^{0}\frac{rad}{s}t + \frac{\pi}{4}) + 100\,mV\cos(10^{+2}\frac{rad}{s}t + \frac{\pi}{4})}$$



*

$V_{in1}(t) = 1mV\cos(10^{-2}t + \frac{\pi}{4}) \xrightarrow{\text{Phasor}} V_{in1} = 10^{-3}e^{+j\frac{\pi}{4}}$

$20\log\left(|H(\omega)|\Big|_{\omega=10^{-2}}\right) = -40\text{dB} \rightarrow |H(\omega)|\Big|_{\omega=10^{-2}} = 10^{-2}$

$\angle H(\omega)\Big|_{\omega=10^{-2}} = 90^{\circ}$ $\rightarrow$ $\boxed{H(\omega)\Big|_{\omega=10^{-2}} = 10^{-2}e^{+j\frac{\pi}{2}}}$ 2

$\boxed{V_{out1} = H(\omega)\Big|_{\omega=10^{-2}} \cdot V_{in1} = 10^{-2}e^{+j\frac{\pi}{2}} \cdot 10^{-3}e^{+j\frac{\pi}{4}} = 10^{-5}e^{+j\frac{3\pi}{4}}}$ 2

$\boxed{V_{out1}(t) = 10^{-5}V\cos(10^{-2}t + \frac{3\pi}{4})}$ 1

* $V_{in2} = 10mv\, e^{j\frac{\pi}{4}}$ or $10^{-2} e^{j\frac{\pi}{4}}$

$20\log\left|H(\omega)\Big|_{\omega=10^0}\right| = 0\,dB \rightarrow |H(10^0)| = 1$, $\measuredangle H(10^0) = 0^0$ $\rightarrow$ $\boxed{H(\omega)\Big|_{\omega=10^0} = 1}$ 2

$\boxed{V_{out2} = V_{in2} \cdot H(\omega)\Big|_{\omega=10^0} = 10^{-2} e^{j\frac{\pi}{4}}}$ 2

$\boxed{V_{out2}(t) = 10^{-2}\cos\left(10^0 t + \frac{\pi}{4}\right) \text{ or } 10mv\cos\left(10^0 t + \frac{\pi}{4}\right)}$ 1

* $V_{in3} = 100\,mV e^{j\frac{\pi}{4}}$ or $10^{-1} e^{j\frac{\pi}{4}}$

$20\log\left|H(\omega)\Big|_{\omega=10^2}\right| = -40\,dB \rightarrow \left|H(\omega)\Big|_{\omega=10^2}\right| = 10^{-2}$, $\measuredangle H(\omega)\Big|_{\omega=10^2} = -90^0$ $\rightarrow$ $\boxed{H(\omega)\Big|_{\omega=10^2} = 10^{-2} e^{-j\frac{\pi}{2}}}$ 2

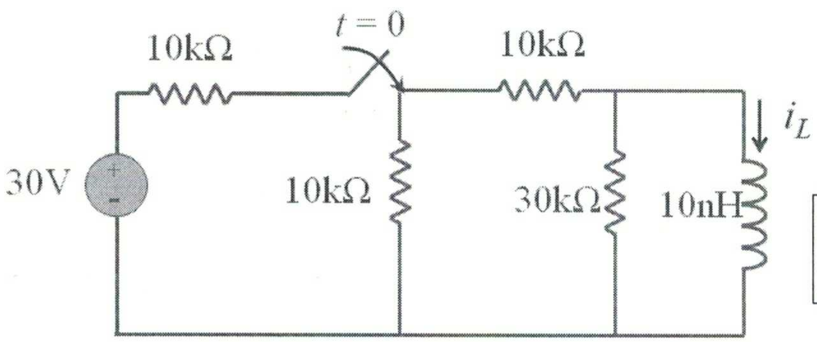
$\boxed{V_{out3} = V_{in3} \cdot H(\omega)\Big|_{\omega=10^2} = 10^{-3} e^{-j\frac{\pi}{4}}}$ 2

$\boxed{V_{out3}(t) = 10^{-3} v\cos\left(10^2 t - \frac{\pi}{4}\right) \text{ or } 1mv\cos\left(10^2 t - \frac{\pi}{4}\right)}$ 1

$V_{out}(t) = V_{out1}(t) + V_{out2}(t) + V_{out3}(t)$

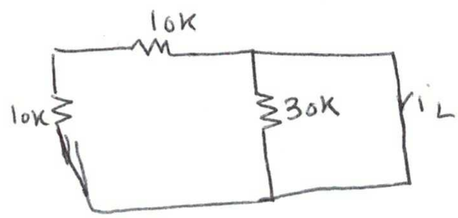
$V_{out}(t)$	$10^{-5}\cos\left(10^{-2}t + \frac{3\pi}{4}\right) + 10^{-2}\cos\left(10^0 t + \frac{\pi}{4}\right) + 10^{-3}\cos\left(10^2 t - \frac{\pi}{4}\right)$

PROBLEM 4: (15 points)
In the circuit below the switch closes at t=0. Find $\boldsymbol{i_L(t)}$.



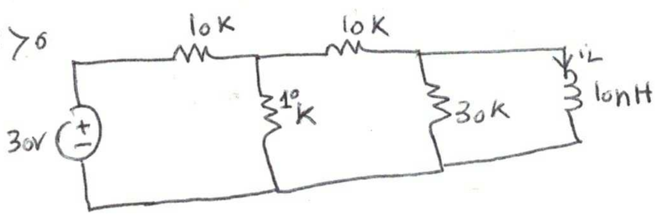
$\boldsymbol{i_L(t)}$	$\left(1-e^{-\frac{t}{10^{-12}}}\right)$ [mA]

t<0

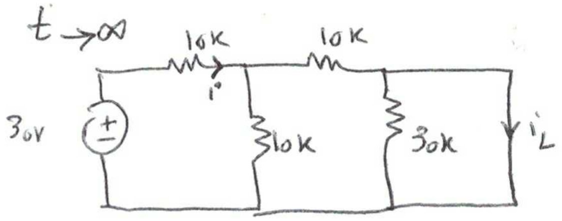


$\boxed{i_L(0)=0}$ 3

t>0



$t\to\infty$

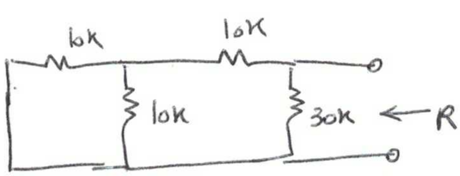


$i = \dfrac{30V}{10K+(10K\,||\,10K)} = 2mA$

$i_L(\infty) = \dfrac{10K}{10K+10K} \times i = 1mA$ 5

$\boxed{i_L(\infty)=1mA}$

τ:



$R = 30K \,||\, \left[\, 10K + (10K\,||\,10K) \right] = 30K\,||\,15K = 10K\Omega$

$\boxed{R=10K\Omega}$ 3

$\tau = \dfrac{L}{R} = \dfrac{10nH}{10K\Omega} = 10^{-12}\,s = 1ps$

$\boxed{\tau = 10^{-12}\,s = 1ps}$

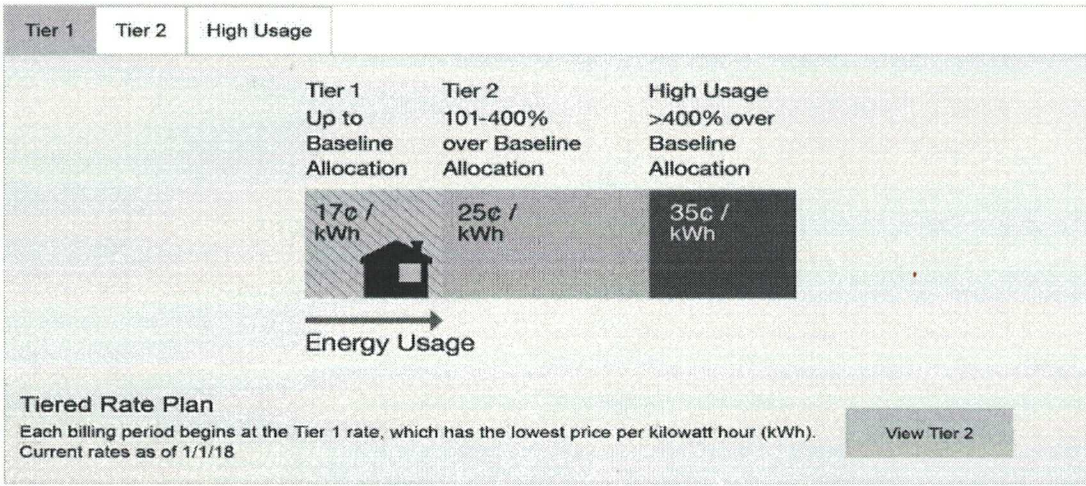
2

$i_L(t) = i_L(\infty) + \left(i_L(0) - i_L(\infty)\right) e^{-\frac{t}{\tau}}$

$i_L(0) = \left(1 - e^{-\frac{t}{10^{-12}}}\right)$ [mA] 2

POBLEM 5: (10 points)

Shown below is the Southern California Edison rate for electricity.



a) How much money does it cost to run a **100 W** light bulb for 1 month? Assume your household is frugal with electricity usage, so you are in Tier 1.

Energy fo 1 month $= P \cdot t = 100\,W \times 24h \times 30 = 24 \times 3\ kWh = \boxed{72\ kWh}$ 3

Cost $= 72\ kWh \times 17 \frac{¢}{kWh} = 1224¢ = \boxed{12.24}\$$

1

Monthly ost of running 1 light bulb	1224¢ or 12.24$

b) If you only use this light bulb, would you still be in the lowest tier? Assume that the tire 1 baseline allocation is **400 kWh** per month.

$\boxed{\text{yes}}$ because $72\,\text{kWh} < 400\,\text{kWh}$ 1

c) What is the minimun number light bulbs you need to use to be in tier 2? Assume that the light bulbs are on 24/7 and tire 1 baseline allocation is **400 kWh** per month.

each light bulb as calculate in section a, consumes 72 kWh per month. In order to be in Tier 2 we need to consume more than 400 kWh.

$n \cdot 72\,\text{kWh} > 400\,\text{kWh} \rightarrow n > \frac{400\,\text{kWh}}{72\,\text{kWh}} = \boxed{5.5}$ 3

The minimum number of light bulbs is $n = \boxed{6}$ 3

(if $n=6$ is provided full grade should be assigned)

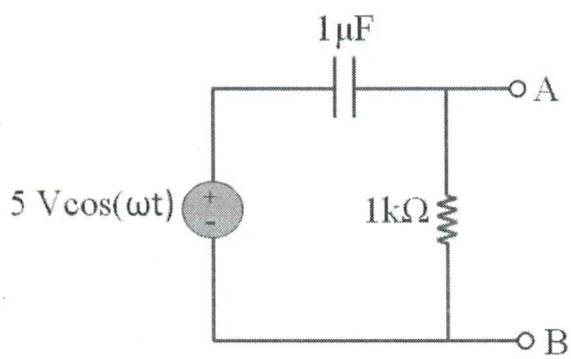
Number of light bulbs	6

PROBLEM 6: (15 points)

Find the Thevenin equivalent circuit at terminals AB by finding $\boldsymbol{V_{oc}}$ and $\boldsymbol{I_{sc}}$. Assume $\omega = 10^3 \frac{rad}{s}$. $\boldsymbol{V_{oc}} = \boldsymbol{V_{AB}}$ (open) and $\boldsymbol{I_{sc}} = \boldsymbol{I_{AB}}$ (short A to B).

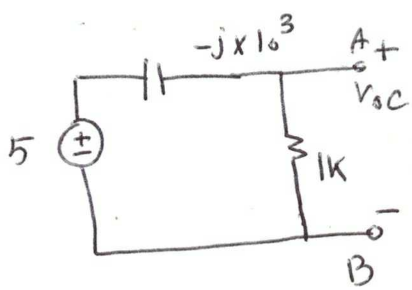
(express your answers for phasors in polar form $re^{j\theta}$)

(express all angles in **radians** not degrees)



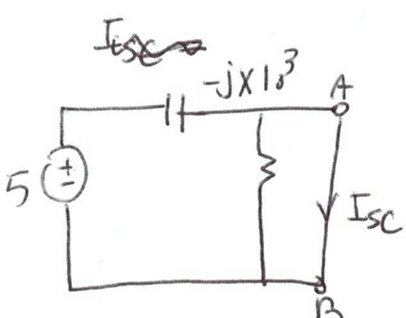
$\boldsymbol{V_{oc}}$	$\frac{5}{2}\sqrt{2}\, e^{j\frac{\pi}{4}}$ [V]
$\boldsymbol{I_{sc}}$	$5mA\, e^{+j\frac{\pi}{2}}$
$\boldsymbol{V_{Th}}$	$\frac{5}{2}\sqrt{2}\, e^{j\pi/4}$ [V]
$\boldsymbol{Z_{Th}}$	$\frac{\sqrt{2}}{2}\, e^{-j\frac{\pi}{4}}$ [kΩ]

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j\,10^3 \times 10^{-6}} = \frac{10^3}{j} = \boxed{-j\,10^3\,\Omega} \quad 3$$



$$V_{oC} = \frac{1K}{1K + (-j \times 10^3)} \cdot 5 = 5 \times \frac{1}{1-j}$$

$$V_{oC} = \frac{5(1+j)}{2} = \boxed{\frac{5}{2} \cdot \sqrt{2} e^{j\frac{\pi}{4}}} \quad 3$$



$$I_{sc} = \frac{5}{-j \times 10^3} = 5 \times 10^{-3} j = \boxed{5mA\, e^{j\frac{\pi}{2}}} \quad 3$$

$$\boxed{V_{th} = V_{oc} = \frac{5}{2}\sqrt{2}\, e^{j\frac{\pi}{4}}} \quad 2$$

$$\boxed{Z_{th} = \frac{V_{oc}}{I_{sc}}}\ 1 = \frac{\frac{5}{2}\sqrt{2} e^{j\frac{\pi}{4}}}{5 \times 10^{-3} e^{j\frac{\pi}{2}}} = \boxed{\frac{\sqrt{2}}{2} \times 10^3 e^{-j\frac{\pi}{4}}\,\Omega}$$

or 3

or $\boxed{\frac{\sqrt{2}}{2} e^{-j\frac{\pi}{4}}\ K\Omega}$