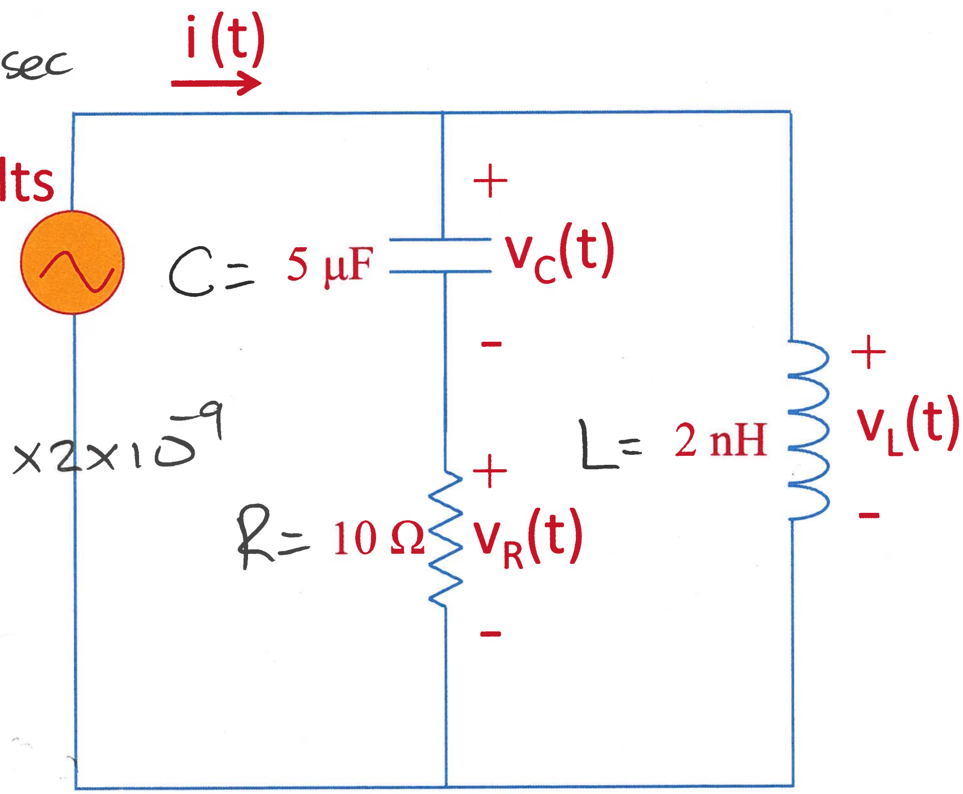
Problem 1: (10 pts)

For the circuit shown below, find $i(t)$, $v_L(t)$, $v_C(t)$ and $v_R(t)$.

$\underbrace{\omega}_{} = 100$ rad/sec

$v_s(t) = 4\cos(100t + 60^\circ)$ volts



$$\overline{V_s} = 4\angle 60^\circ \text{ V}$$

$$\overline{Z}_{eq} = \left(\frac{1}{j(100)(5\times10^{-6})} + 10\right) \| \, j(100)\times 2\times10^{-9}$$

$$= 2\times10^{-7}\angle 90^\circ \ \Omega$$

$$\overline{I} = \frac{\overline{V_s}}{\overline{Z}_{eq}} = 2\times10^{7}\angle -30^\circ \text{ A}$$

$$i(t) = 2\times10^{7}\cos(100t - 30^\circ) \text{ A}$$

$$\overline{V}_c = \overline{V}_s \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = 4 \angle 59.71^\circ \text{ V}$$

$$V_c(t) = 4\cos(100t + 59.71^\circ) \text{ V}$$

$$V_R = V_s \frac{R}{R + \frac{1}{j\omega C}} = 0.02 \angle 149.71^\circ \text{ V}$$

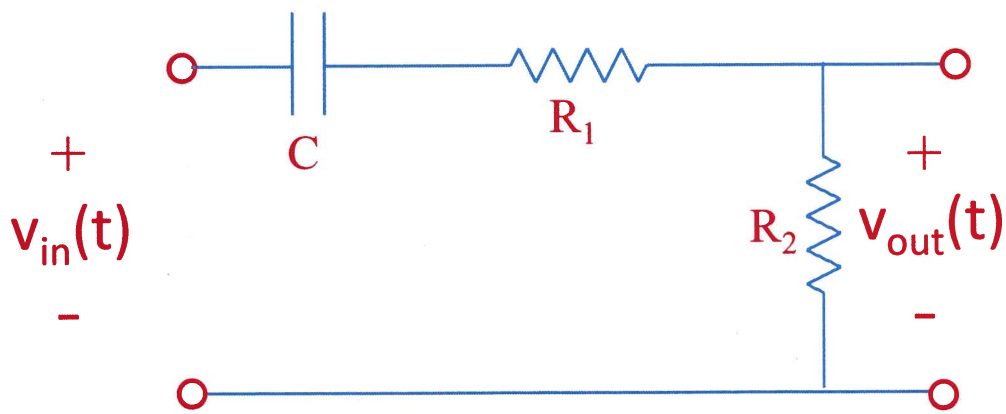
$$V_R(t) = 0.02 \cos(100t + 149.71^\circ) \text{ V}$$

$$V_L(t) = V_s(t) = 4\cos(100t + 60^\circ) \text{ V}$$

Problem 2: (10 pts)

Determine the type of the filter shown below based on C, R_1 and R_2.

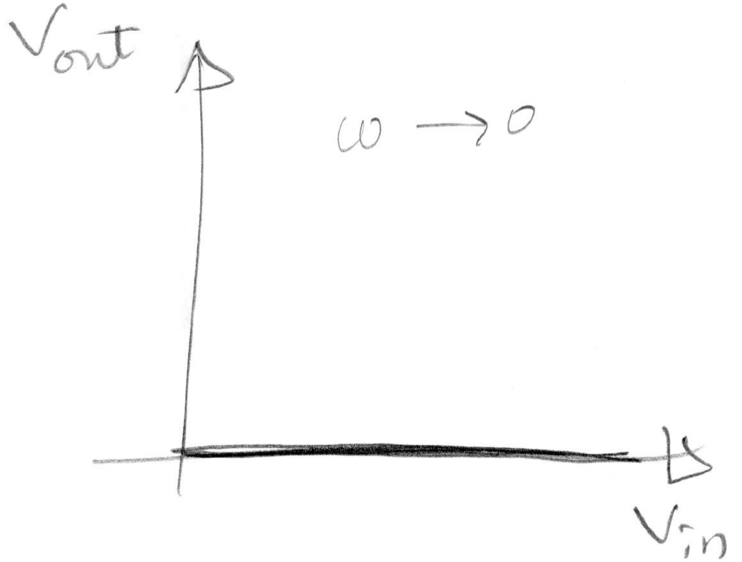
Plot $V_{out}(t)$ versus $V_{in}(t)$ for $\omega \to 0$ and $\omega \to \infty$.

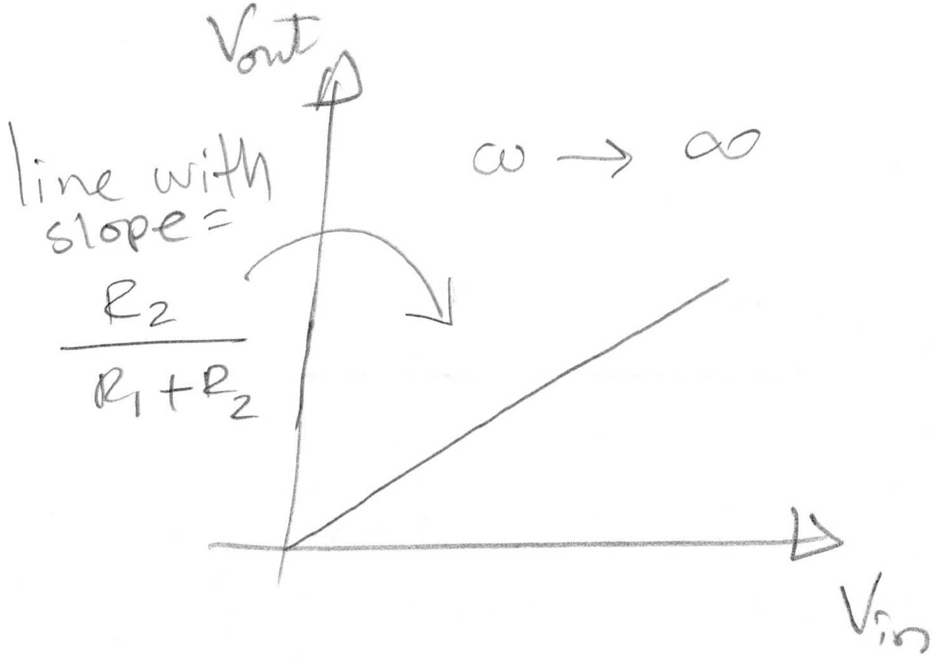


$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2 + \frac{1}{j\omega C}} = \frac{j\omega R_2 C}{j\omega R_1 C + j\omega R_2 C + 1}$$

$$\lim_{\omega \to 0} \frac{V_{out}}{V_{in}} = 0$$

$$\lim_{\omega \to \infty} \frac{V_{out}}{V_{in}} \overset{L'H}{=} \frac{jR_2C}{jR_1C + jR_2C} = \frac{R_2}{R_1 + R_2}$$



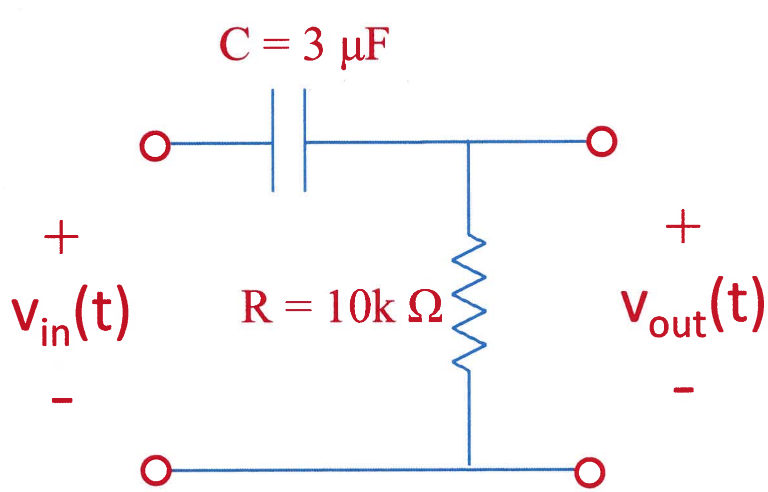


Problem 3a: (10 pts)

Find the transfer function $H(\omega)$, $|H(\omega)|$ and $\angle H(\omega)$.

Plot $|H(\omega)|$ on linear-linear and log-log scales.

Plot $\angle H(\omega)$ on linear-log scales.



$$H(\omega) = \frac{R}{R + \frac{1}{j\omega C}}$$

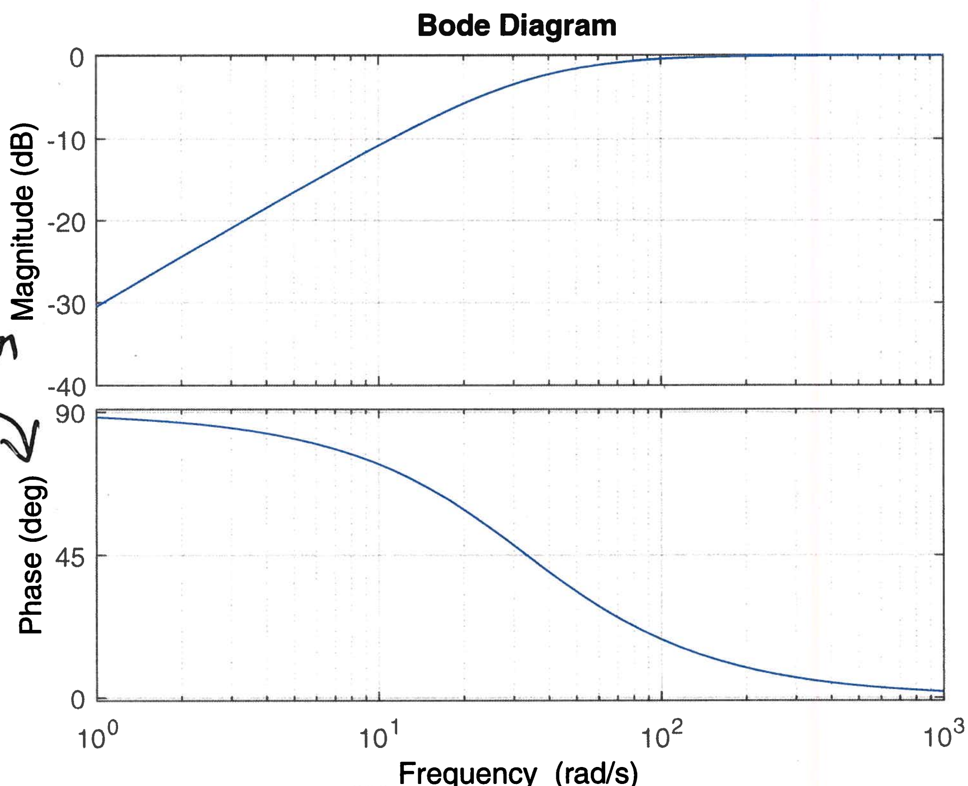
$$= \frac{j\omega RC}{j\omega RC + 1}$$

$$|H(\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

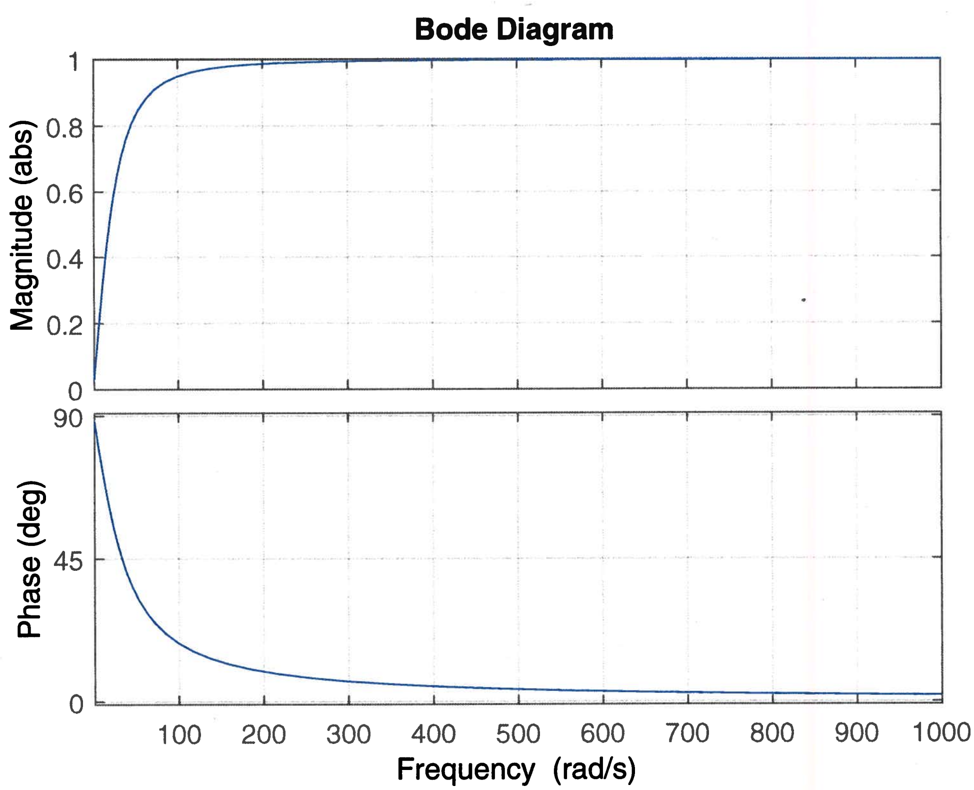
$$\angle H(\omega) = 90^\circ - \tan^{-1}(\omega RC)$$

For all plots, rotate view to have frequency axis horizontal.

vertical



Horizontal

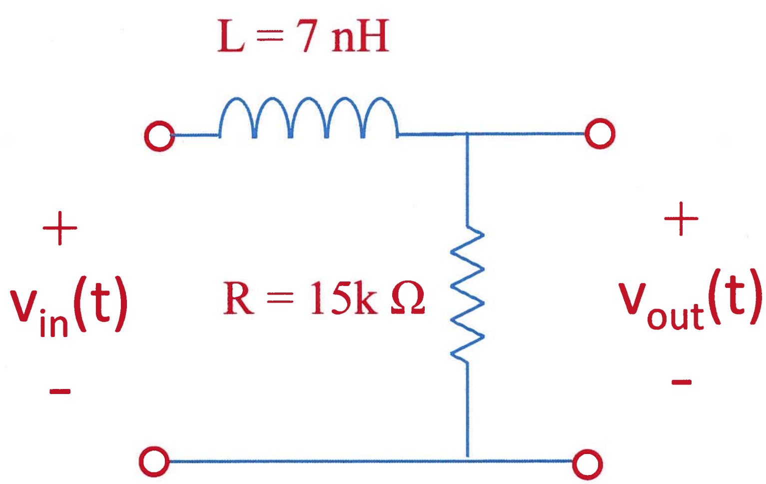

Bode Diagram
Magnitude (abs)
1
0.8
0.6
0.4
0.2
0
Phase (deg)
90
45
0
100
200
300
400
500
600
700
800
900
1000
Frequency (rad/s)

Problem 3b: (10 pts)

Find the transfer function $H(\omega)$, $|H(\omega)|$ and $\angle H(\omega)$.

Plot $|H(\omega)|$ on linear-linear and log-log scales.

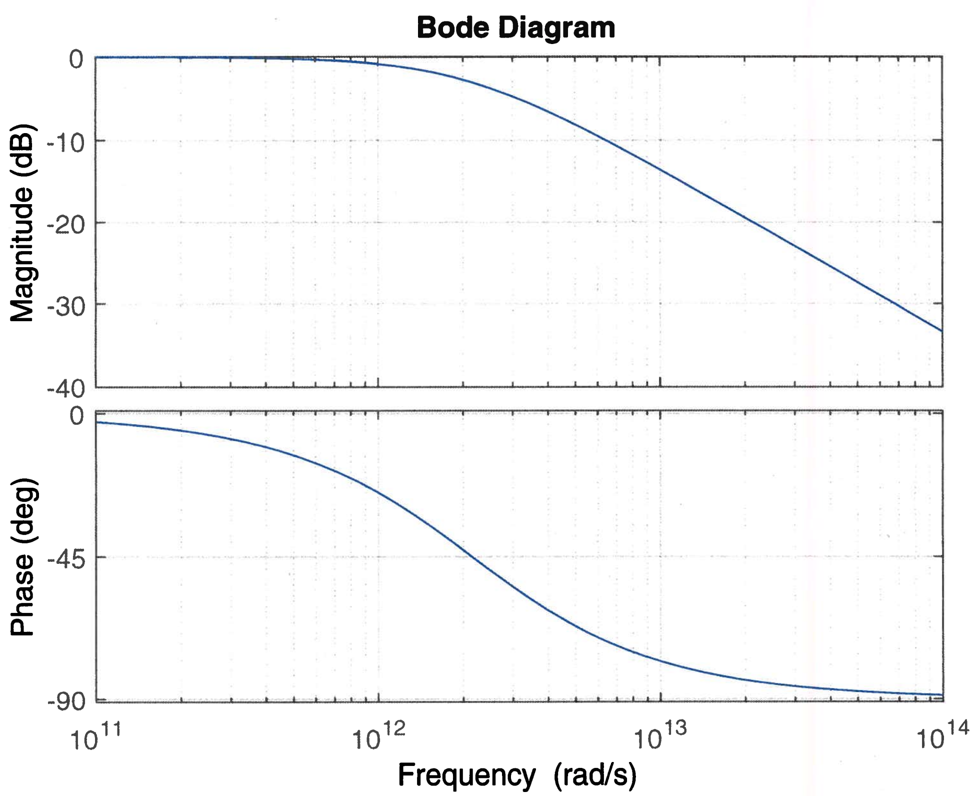
Plot $\angle H(\omega)$ on linear-log scales.

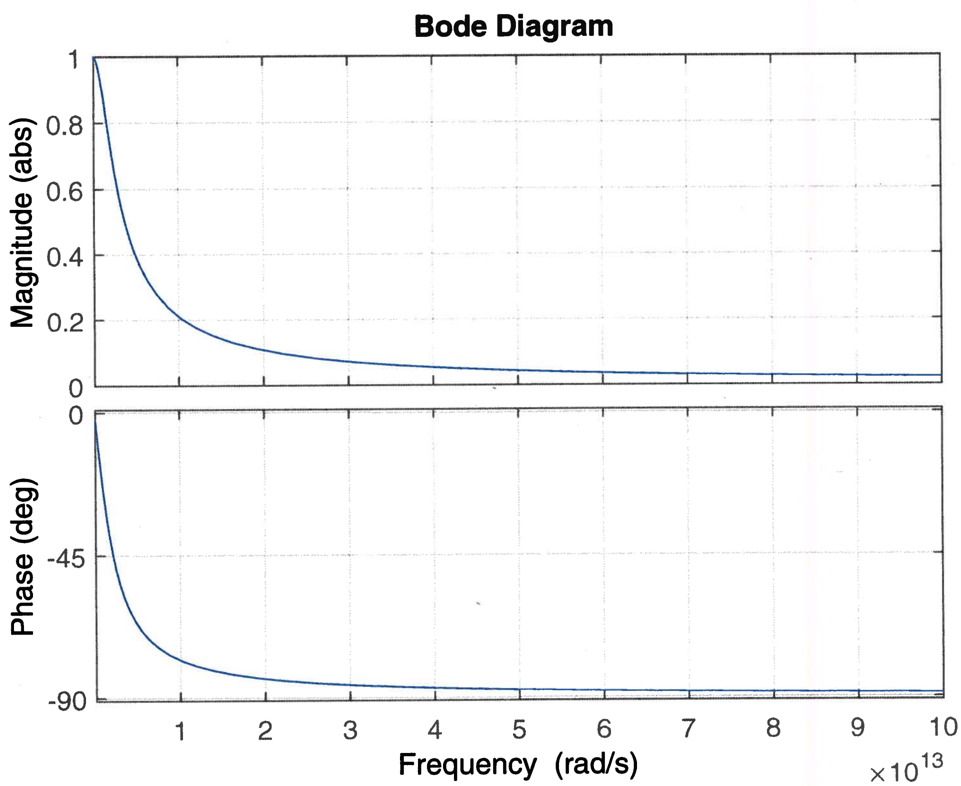


$$H(\omega) = \frac{R}{R + j\omega L}$$

$$|H(\omega)| = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

$$\angle H(\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$


Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
Phase (deg)
0
-45
-90
10^{11}
10^{12}
10^{13}
10^{14}
Frequency (rad/s)



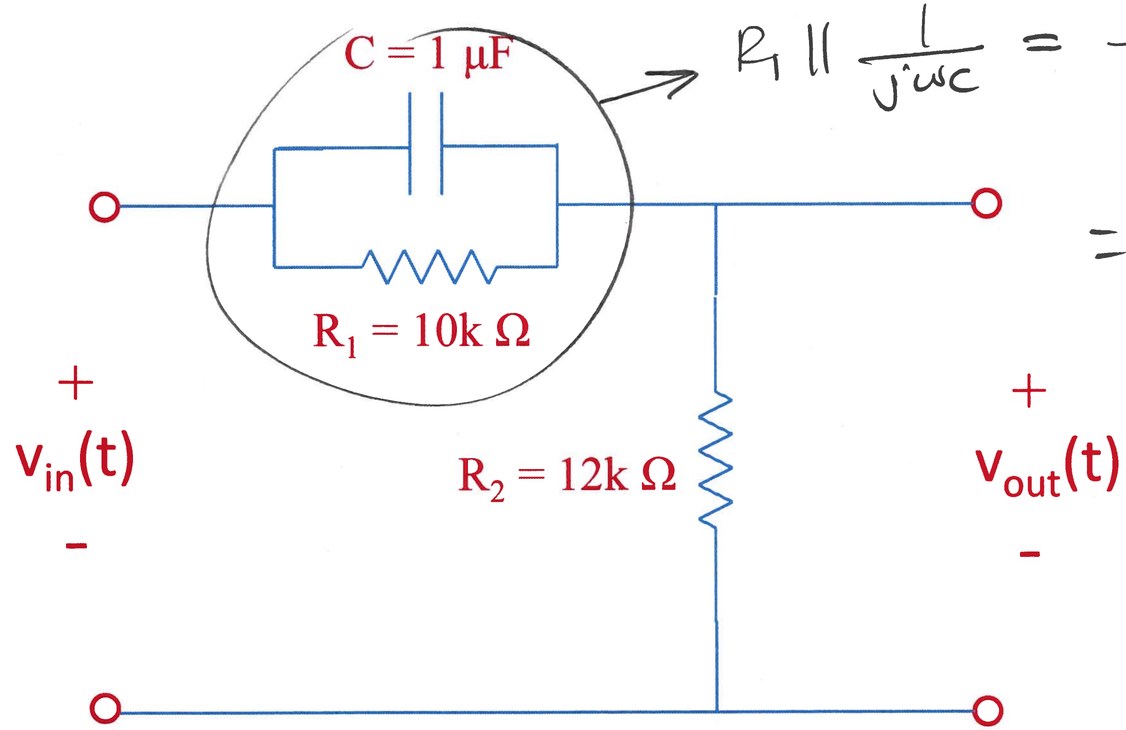
Bode Diagram
Magnitude (abs)
1
0.8
0.6
0.4
0.2
0
Phase (deg)
0
-45
-90
1
2
3
4
5
6
7
8
9
10
Frequency (rad/s)
×10^13

Problem 4: (10 pts)

Find the transfer function $H(\omega)$, $|H(\omega)|$ and $\angle H(\omega)$.

Plot $|H(\omega)|$ on linear-linear and log-log scales.

Plot $\angle H(\omega)$ on linear-log scales.



$$R_1 \parallel \frac{1}{j\omega C} = \frac{R_1 \frac{1}{j\omega C}}{R_1 + \frac{1}{j\omega C}} = \frac{\frac{R_1}{j\omega C}}{\frac{j\omega R_1 C + 1}{j\omega C}} = \frac{R_1}{\cancel{j\omega C}} \cdot \frac{\cancel{j\omega C}}{1 + j\omega R_1 C}$$

$$= \frac{R_1}{1 + j\omega R_1 C}$$

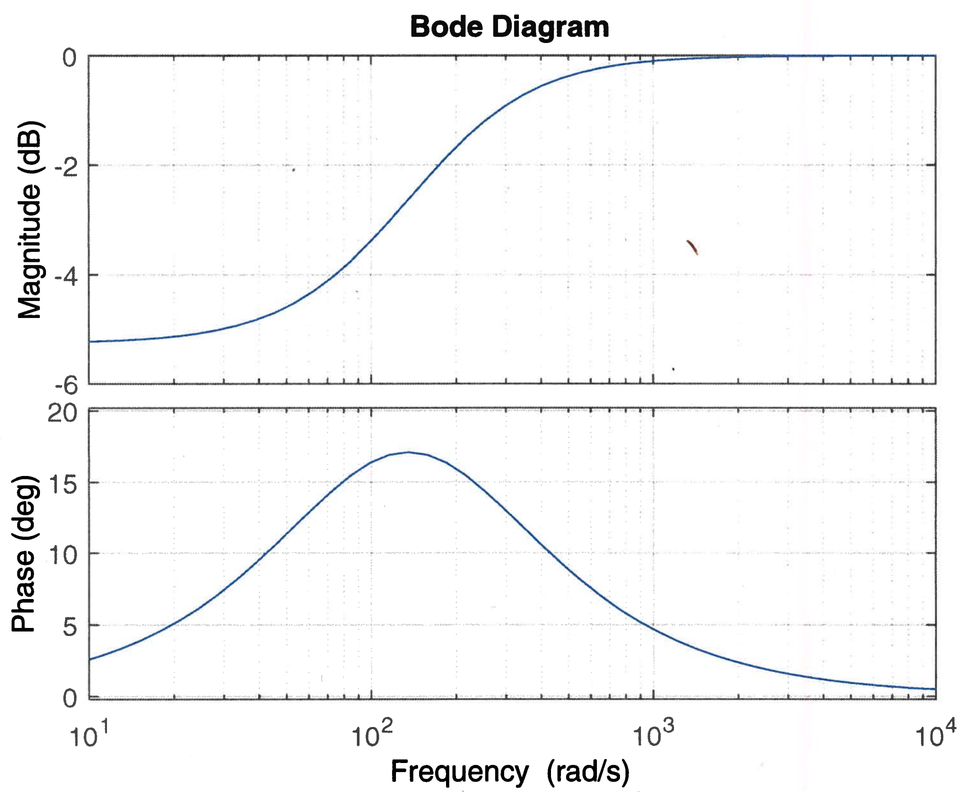
$$H(\omega) = \frac{R_2}{R_2 + \frac{R_1}{1+j\omega R_1 C}}$$

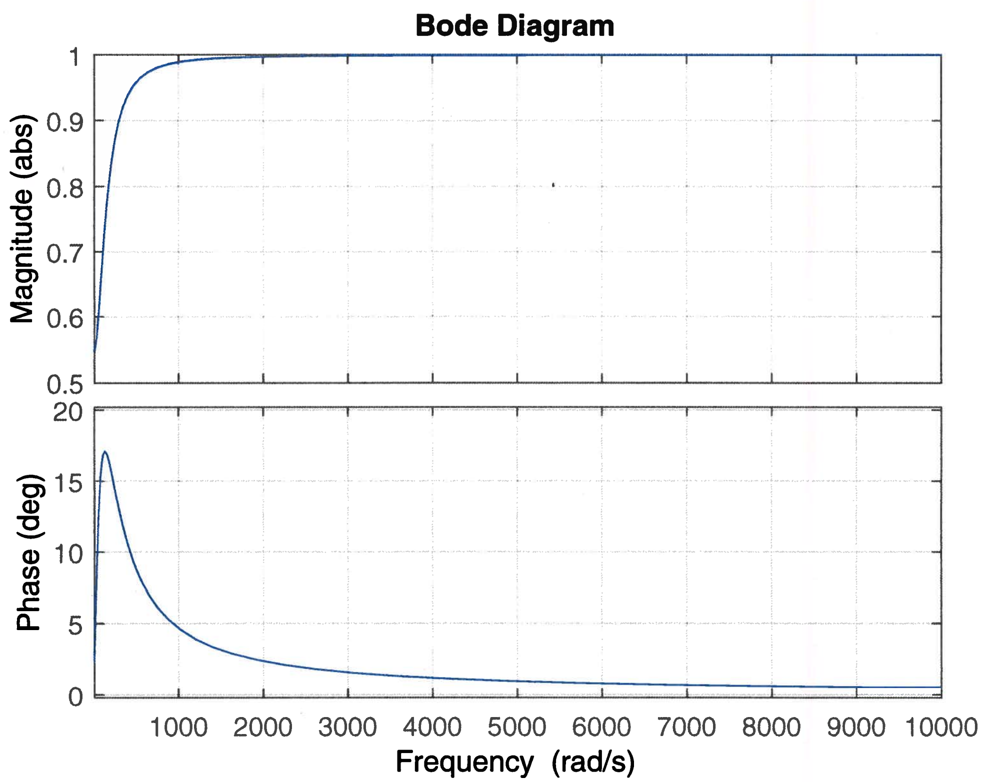
$$= \frac{R_2}{\frac{R_2 + j\omega R_1 R_2 C + R_1}{1+j\omega R_1 C}}$$

$$= \frac{R_2 + j\omega R_1 R_2 C}{R_1 + R_2 + j\omega R_1 R_2 C}$$

$$|H(\omega)| = \frac{\sqrt{(R_2)^2 + (\omega R_1 R_2 C)^2}}{\sqrt{(R_1+R_2)^2 + (\omega R_1 R_2 C)^2}}$$

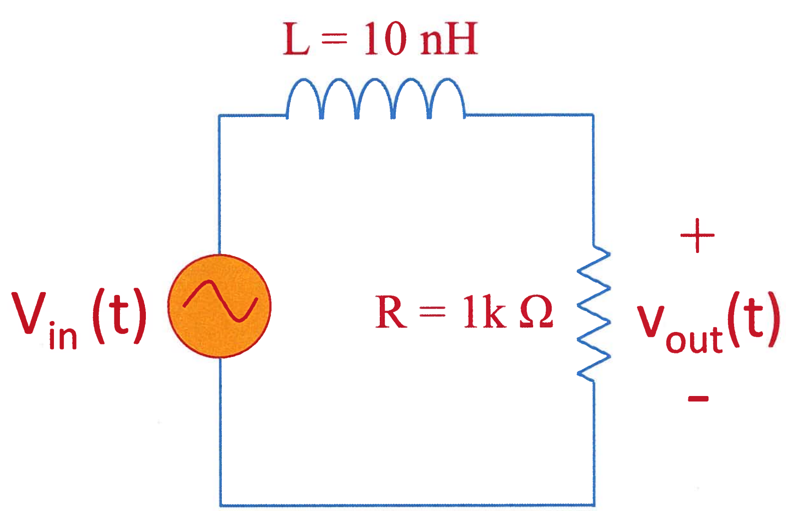
$$\angle H(\omega) = \tan^{-1}\left(\frac{\omega R_1 R_2 C}{R_2}\right) - \tan^{-1}\left(\frac{\omega R_1 R_2 C}{R_1+R_2}\right)$$


Bode Diagram
Magnitude (dB)
0
-2
-4
-6
Phase (deg)
20
15
10
5
0
10^1
10^2
10^3
10^4
Frequency (rad/s)


Bode Diagram
Magnitude (abs)
1
0.9
0.8
0.7
0.6
0.5
20
15
10
5
0
Phase (deg)
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Frequency (rad/s)

Problem 5: (10pts)

For f = 1, 10, 100, 1k, 10k, and 100k Hz, find the output voltage as $V_{out}(t) = A\cos(2\pi f t + \phi)$ where ϕ is the phase if the input voltage is $V_{in}(t) = 10\cos(2\pi f t + \pi/3)$



$$\overline{V_{in}} = 10e^{j\pi/3}$$

$$\overline{V_{out}} = \overline{V_{in}}\frac{R}{R + j\omega L}$$

$$= 10e^{j\pi/3}\frac{1\times10^3}{1\times10^3 + j(2\pi f)\times10\times10^{-9}}$$

or $\pi/3$ rad

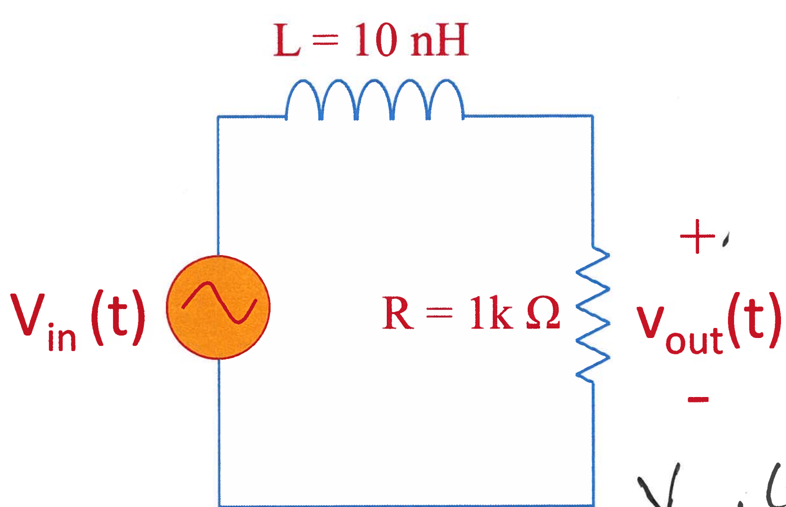
f (Hz)	$\bar{V}_{out}$	$V_{out}(t) =$
1	use the $\bar{V}_{out}$ equation on the previous page	$V_1(t) = 10\cos(2\pi t + 60°)$
10		$V_2(t) = 10\cos(20\pi t + 60°)$
100		$V_3(t) = 10\cos(200\pi t + 60°)$
1K		$V_4(t) = 10\cos(2{,}000\pi t + 60°)$
10K		$V_5(t) = 10\cos(20{,}000\pi t + 60°)$
100K		$V_6(t) = 10\cos(200{,}000\pi t + 60°)$

Problem 6: (10pts)

Find the output voltage as

$V_{out}(t) = A\cos(2\pi ft + \phi)$ where ϕ is the phase if the input voltage is

$V_{in}(t) = 10\Sigma_i \cos(2\pi f_i t + \pi/3)$, f_i = 1, 10, 100, 1k, 10k, and 100k Hz



Linear circuit → the output is the superposition of the input signals.

From Problem 5,

$$V_{out}(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t)$$

Problem 7: (10pts)

Sketch the Bode plot (magnitude only) for the following transfer function

$H(\omega) = 1/((1+j\omega\tau) \cdot (1+j\omega\tau))$