EECS/CSE 70A Network Analysis I

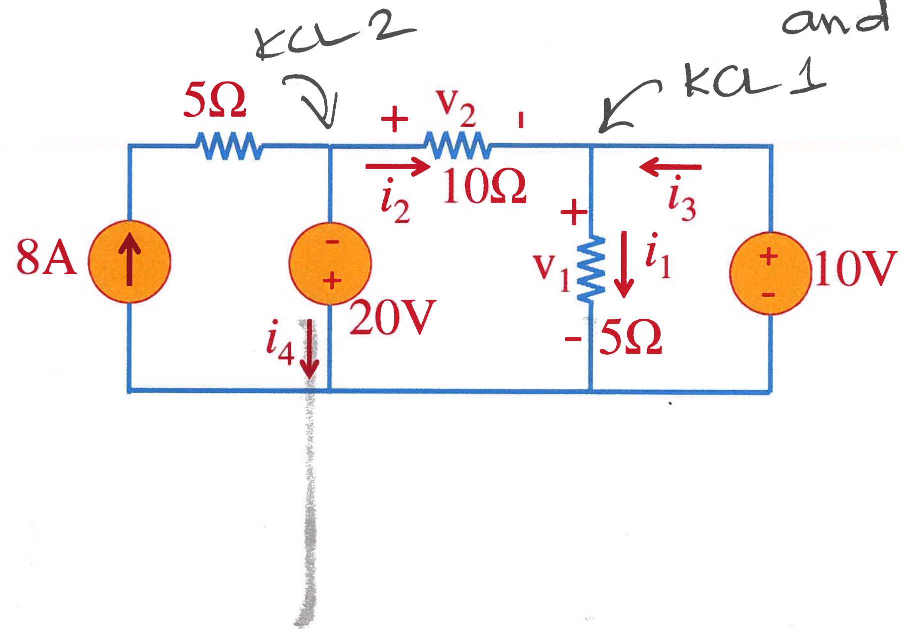
Homework #3 Solutions

Due on or before

4/26/2018, Thursday at 10:00AM

(You can submit your homework in any of the discussion sessions on either Tuesday 4/24 or Thursday 4/26)

Problem 1: (KCL, KVL, Ohm's Law) Find currents i_1, i_2, i_3, *and* i_4 (10pts.) and voltages V_1 and V_2.



$\boxed{V_1 = 10V}$

$i_1 = \frac{V_1}{5} = 10/5 \Rightarrow \boxed{i_1 = 2A}$

Applying KVL @ middle loop,

$+20 + V_2 + V_1 = 0 \Rightarrow +20 + V_2 + 10 = 0$

$\Rightarrow \boxed{V_2 = -30V}$ $\quad i_2 = \frac{V_2}{10} \Rightarrow \boxed{i_2 = -3A}$

Applying KCL at top right node,

$i_2 + i_3 = i_1 \Rightarrow i_3 = i_1 - i_2 = 2 + 3$

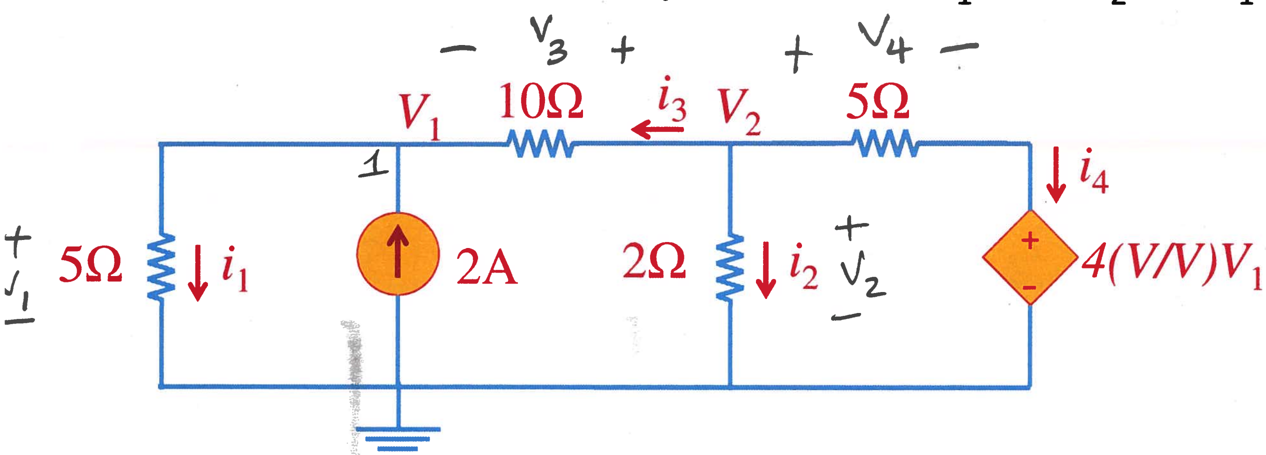
$\Rightarrow \boxed{i_3 = 5A}$

KCL at middle left node,

$8 = i_2 + i_4 \Rightarrow i_4 = 8 - i_2 = 8 + 3$

$\Rightarrow \boxed{i_4 = 11A}$

Problem 2: Use nodal analysis and find V_1 ~~and~~ V_2, V_3, and V_4 and i_1 through i_4 (10pts.)



@ node 1, $\frac{V_1 - 0}{5} - 2 + \frac{V_1 - V_2}{10} = 0$

Multiply by 10,

$2V_1 - 20 + V_1 - V_2 = 0 \Rightarrow 3V_1 - V_2 = 20$ —— ①

@ node 2, $\frac{V_2 - V_1}{10} + \frac{V_2 - 0}{2} + \frac{V_2 - 4V_1}{5} = 0$

Multiply by 10,

$V_2 - V_1 + 5V_2 + 2V_2 - 8V_1 = 0$

$-9V_1 + 8V_2 = 0$ —— ②

Multiply ① by 3,

$9V_1 - 3V_2 = 60$ —— ⊛ ← corrected

② + ⊛ $\Rightarrow +5V_2 = 60$

$\Rightarrow \boxed{V_2 = 12V}$

From ①, $V_1 = \frac{20 + V_2}{3}$

$\Rightarrow \boxed{V_1 = 10.67V}$

$i_1 = \frac{V_1}{5} \Rightarrow \boxed{i_1 = 2.13A}$

$i_2 = \frac{V_2}{2} \Rightarrow \boxed{i_2 = 6A}$

$V_3 = V_2 - V_1 \Rightarrow$

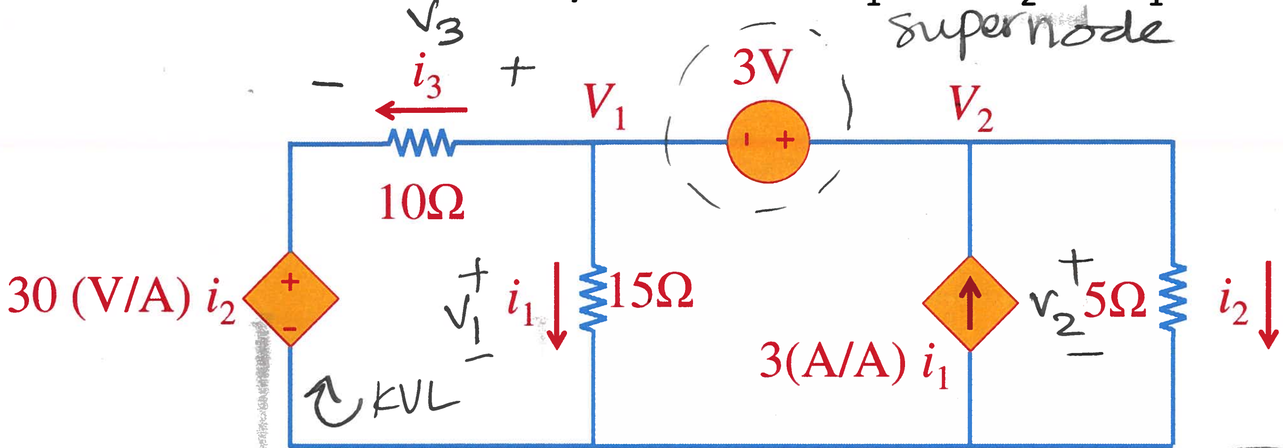
$\Rightarrow \boxed{V_3 = +1.33V}$

$i_3 = \frac{V_3}{10} \Rightarrow \boxed{i_3 = 0.13A}$

$\boxed{V_4} = V_2 - 4V_1 = \boxed{-30.68V}$

$\Rightarrow \boxed{i_4 = V_4/5 = -6.14A}$

Problem 3: Use nodal analysis and find V_1 ~~and~~ V_2 and i_1 through i_3 (10pts.) and V_3



$-V_1 - 12V_2 = 0$

$V_1 + 12V_2 = 0$ —— (*)

@ supernode,

$V_2 = V_1 + 3$

$-V_1 + V_2 = 3$ —— (**)

KCL @ supernode,

$$\frac{V_1 - 30i_2}{10} + \frac{V_1}{15} - 3i_1 + \frac{V_2}{5} = 0 \quad \text{—— (1)}$$

$i_1 = \frac{V_1}{15}$ and $i_2 = \frac{V_2}{5}$

put i_1 and i_2 in (1),

$$\frac{V_1 - 6V_2}{10} + \frac{V_1}{15} - \frac{V_1}{5} + \frac{V_2}{5} = 0$$

Multiply by 30

$3V_1 - 18V_2 + 2V_1 - 6V_1 + 6V_2 = 0$

(*) + (**),

$13V_2 = 3 \Rightarrow \boxed{V_2 = 3/13V}$

From (*), $V_1 = -12V_2$

$\Rightarrow \boxed{V_1 = -36/13V}$

$i_1 = \frac{V_1}{15} \Rightarrow \boxed{i_1 = -12/65A}$

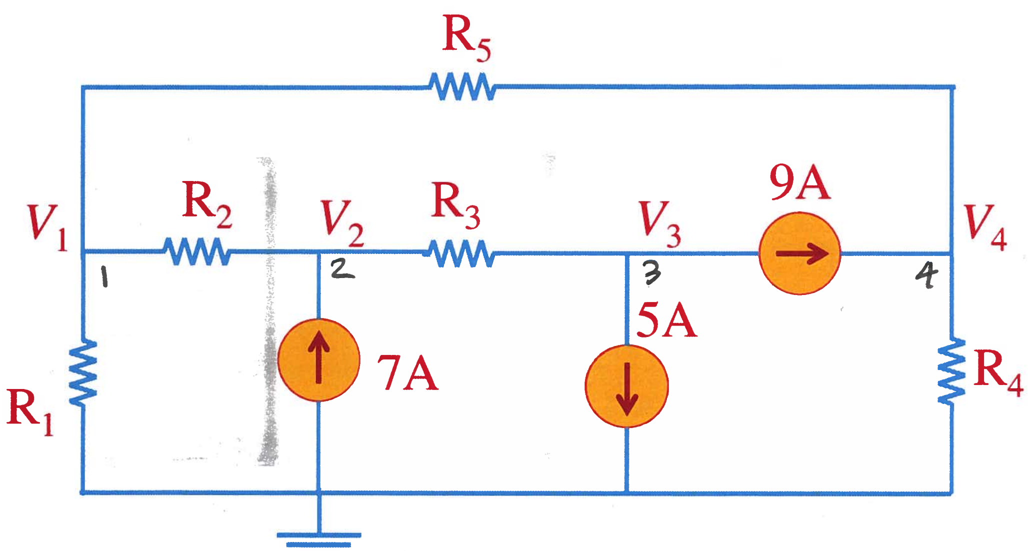
$i_2 = \frac{V_2}{5} \Rightarrow \boxed{i_2 = 3/65A}$

KVL @ left loop, $-30i_2 - V_3 + V_1 = 0$

$\Rightarrow V_3 = V_1 - 30i_2 = -36/13 - 30 \cdot 3/65$

$\Rightarrow \boxed{V_3 = -54/13V}$ $\quad i_3 = V_3/10 \Rightarrow \boxed{i_3 = -54/130A}$

Problem 4: Write all nodal voltage equations and put them in the matrix form (you do not need to solve for voltages V_1 through V_4) (10pts.)



@ node 1, $\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_4}{R_5} = 0$

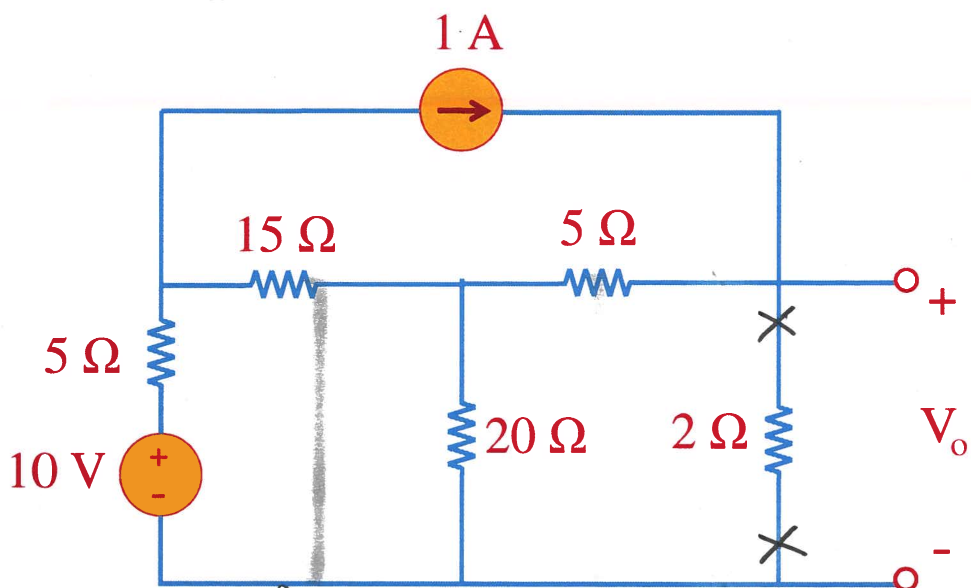
@ node 2, $\frac{V_2 - V_1}{R_2} - 7 + \frac{V_2 - V_3}{R_3} = 0$

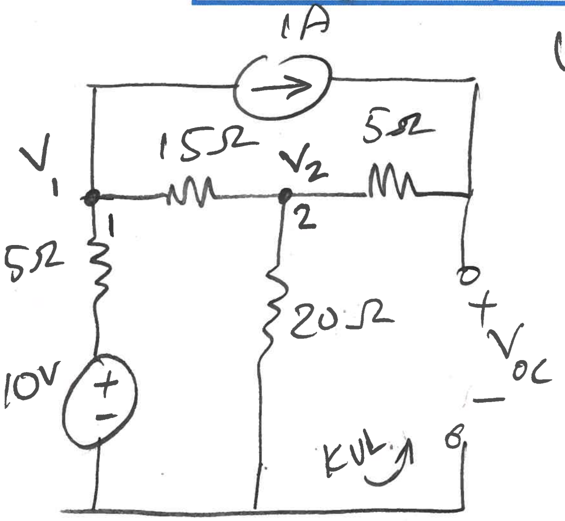
@ node 3, $\frac{V_3 - V_2}{R_3} + 5 + 9 = 0$

@ node 4, $-9 + \frac{V_4}{R_4} + \frac{V_4 - V_1}{R_5} = 0$

$$\begin{bmatrix} \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}\right) & \left(\frac{-1}{R_2}\right) & 0 & \left(\frac{-1}{R_5}\right) \\ \left(\frac{-1}{R_2}\right) & \left(\frac{1}{R_2}+\frac{1}{R_3}\right) & \left(\frac{-1}{R_3}\right) & 0 \\ 0 & \left(\frac{-1}{R_3}\right) & \left(\frac{1}{R_3}\right) & 0 \\ \left(\frac{-1}{R_5}\right) & 0 & 0 & \left(\frac{1}{R_4}+\frac{1}{R_5}\right) \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \\ -14 \\ 9 \end{bmatrix}$$

Problem 5: Find V_o using Thévenin theorem (10pts.)





Using nodal analysis,

@ node 1,

$$\frac{V_1 - 10}{5} + \frac{V_1 - V_2}{15} + 1 = 0$$

Multiply by 15,

$$3V_1 - 30 + V_1 - V_2 + 15 = 0$$

$$4V_1 - V_2 = +15 \quad (1)$$

@ node 2,

$$\frac{V_2 - V_1}{15} + \frac{V_2}{20} - 1 = 0$$

Multiply by 60,

$$4V_2 - 4V_1 + 3V_2 - 60 = 0$$

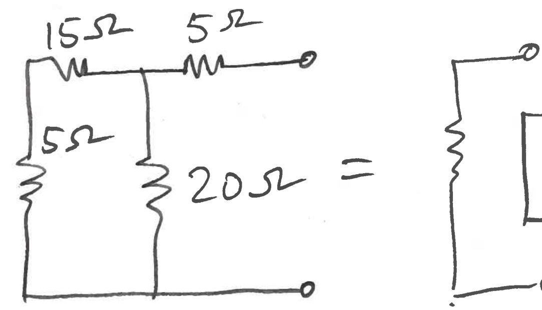
$$-4V_1 + 7V_2 = 60 \quad (2)$$

(1) + (2)

$$6V_2 = 75 \Rightarrow V_2 = 12.5V$$

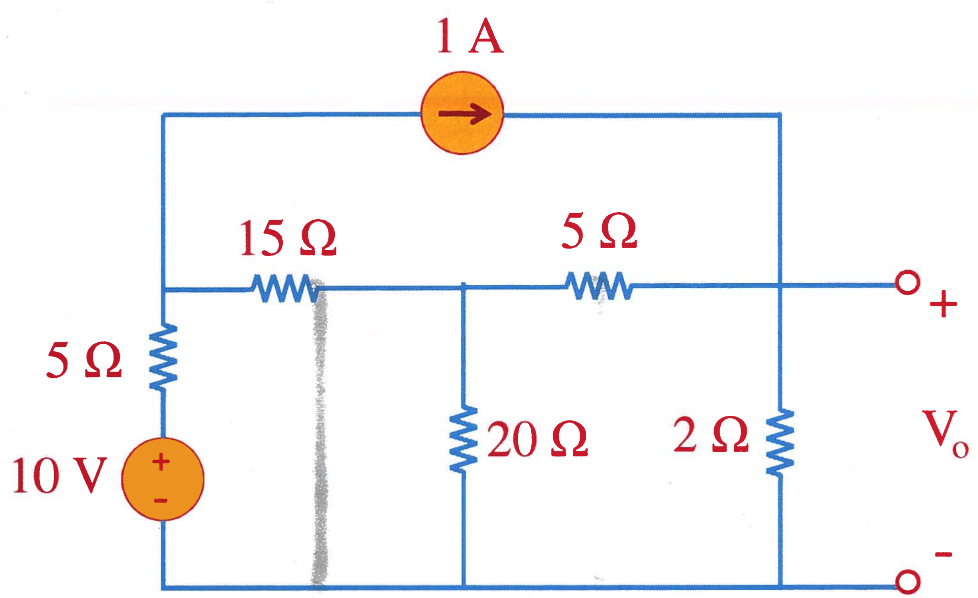
Using KVL, $-V_{oc} + 5 + 12.5 = 0$

$\Rightarrow$ $V_{oc} = 17.5V$



$15\Omega = R_{Th}$

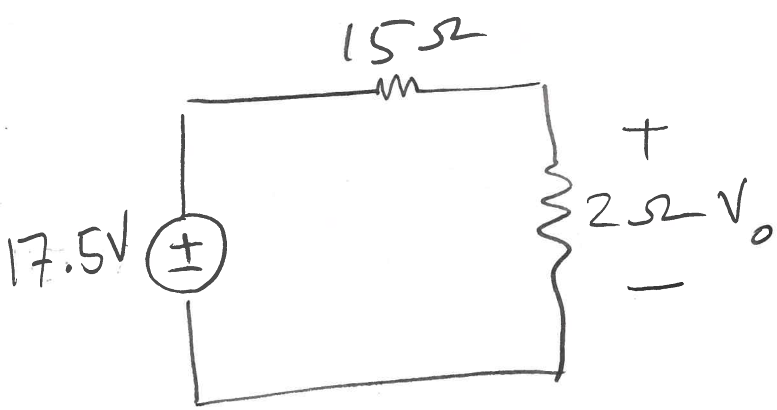
Problem 5: Find V_o using Thévenin theorem (10pts.)



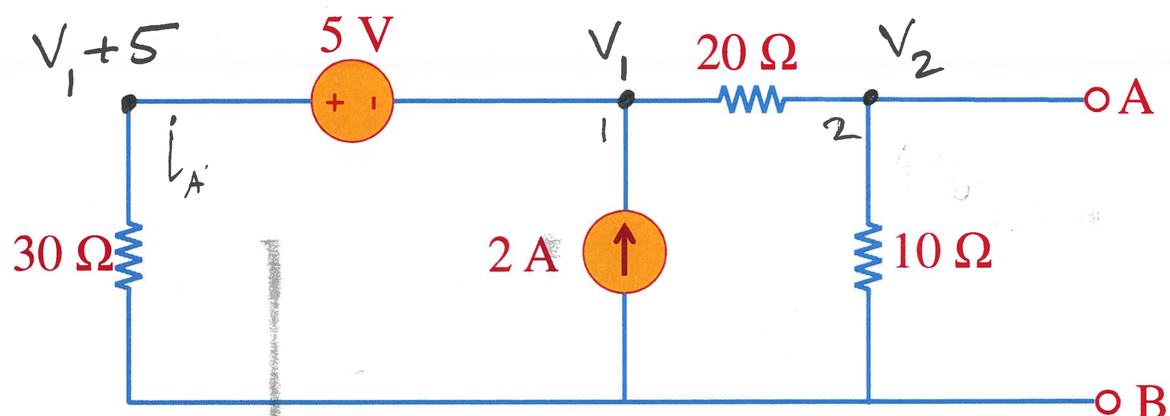
Using voltage divider,

$$V_o = 17.5 \frac{2}{15+2}$$

$$\Rightarrow \boxed{V_o = 2.06V}$$



Problem 6: Find both the Thevenin and Norton equivalent circuits at terminals AB. (10pts.)



$i_A = \frac{V_1+5}{30}$

Applying nodal analysis @ node 1,

$\frac{V_1+5}{30} - 2 + \frac{V_1 - V_2}{20} = 0$

Multiply by 60,

$2V_1 + 10 - 120 + 3V_1 - 3V_2 = 0$

$5V_1 - 3V_2 = 110$ —— ①

At node 2,

$\frac{V_2 - V_1}{20} + \frac{V_2}{10} = 0$

Multiply by 20,

$V_2 - V_1 + 2V_2 = 0$

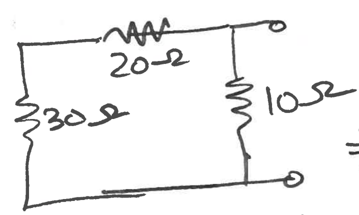
$-V_1 + 3V_2 = 0$ —— ②

Multiply ② by 5 and add to ①

$12V_2 = 110 \Rightarrow V_2 = \frac{55}{6} V = V_{oc}$

$\Rightarrow$ $V_{Th} = 55/6$ V

To Find R_{Th}, use



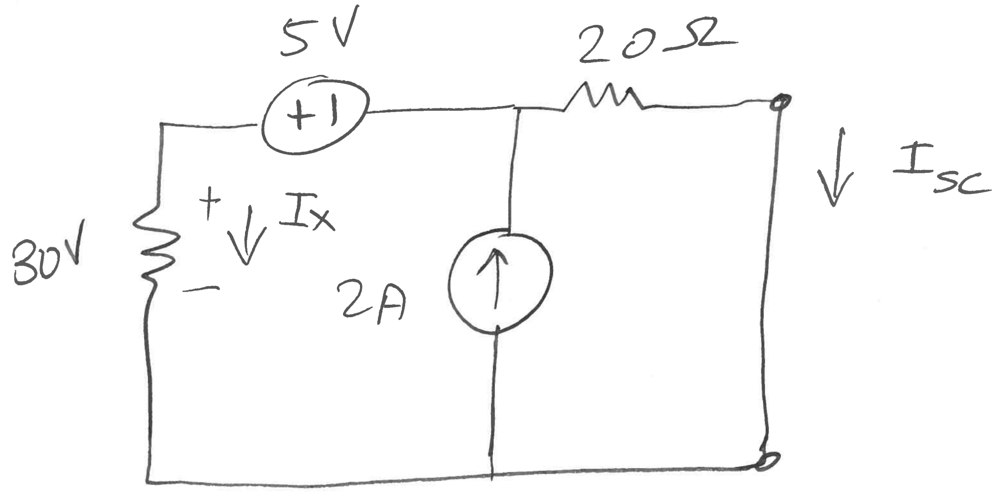
$R_{Th} = 50 \| 10 = \frac{500}{60}$

$\Rightarrow$ $R_{Th} = 25/3\ \Omega = R_N$

$I_{sc} = \frac{V_{Th}}{R_{Th}} = \frac{55}{6} \cdot \frac{3}{25} = \frac{165}{150}$

$\Rightarrow$ $I_N = 11/10$ A

Problem 6. Alt. method to find R_{Th}



$$-30 I_x + 5 + 20 I_{sc} = 0$$

$$\Rightarrow 4 I_{sc} - 6 I_x = -1 \quad \text{—— (1)}$$

$$2 = I_{sc} + I_x \Rightarrow I_x = 2 - I_{sc} \quad \text{—— (2)}$$

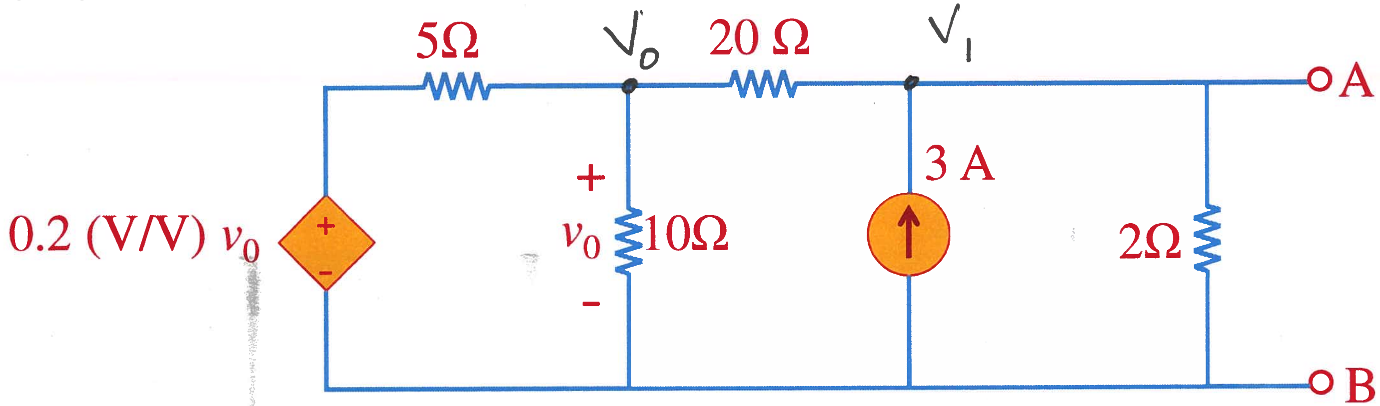
(2) in (1)

$$4 I_{sc} - 6(2 - I_{sc}) = -1$$

$$10 I_{sc} = 11 \Rightarrow \boxed{I_{sc} = 11/10 \text{ A}}$$

$$\boxed{R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{55}{6} \cdot \frac{10}{11} = 25/3\ \Omega = R_N}$$

Problem 7: Find both the Thevenin and Norton equivalent circuits at terminals AB. (10pts.)



Nodal analysis @ V_o,

$$\frac{V_o - 0.2V_o}{5} + \frac{V_o}{10} + \frac{V_o - V_1}{20} = 0$$

$0.31\,V_o - 0.05\,V_1 = 0$ —— (1)

@ V_1,

$$\frac{V_1 - V_o}{20} - 3 + \frac{V_1}{2} = 0$$

$-0.05V_o + 0.55V_1 = 3$ —— (2)

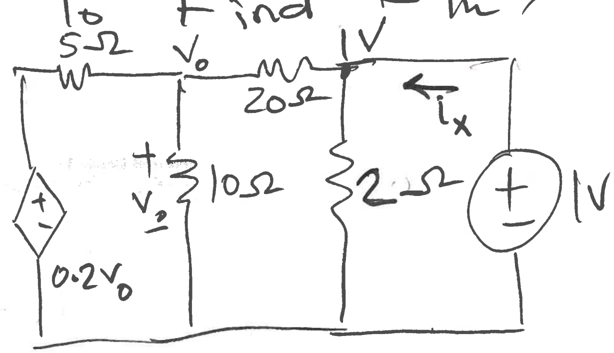
Multiply (1) by 11

$3.41V_o - 0.55\,V_1 = 0$ —— (*)

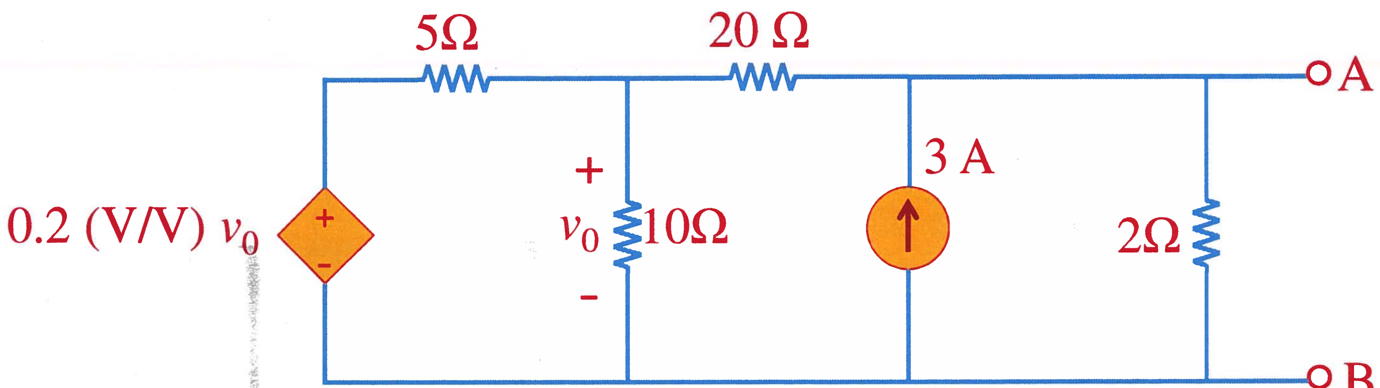
(2) + (*), $3.36\,V_o = 3 \Rightarrow V_o = 0.89\,V$

From (1), $V_1 = 5.52\,V = V_{oc} = V_{Th}$

To Find R_{Th},



Problem 7: Find both the Thevenin and Norton equivalent circuits at terminals AB. (10pts.)



Nodal analysis @ V_o,

$$\frac{V_o - 0.2V_o}{5} + \frac{V_o}{10} + \frac{V_o - 1}{20} = 0$$

Multiply by 20,

$$4V_o - 0.8V_o + 2V_o + V_o - 1 = 0$$

$$V_o = 0.16\,V$$

@ 1V node,

$$\frac{1 - V_o}{20} + \frac{1}{2} - i_x = 0$$

$$\Rightarrow i_x = 0.542\,A$$

$$R_{Th} = \frac{1\,V}{0.542} = 1.85\,\Omega = R_N$$

$$I_{sc} = \frac{V_{Th}}{R_N} = 2.93\,A$$