

EECS/CSE 70A Network Analysis I

Homework #3 Solutions

Due on or before

4/26/2018, Thursday at 10:00AM

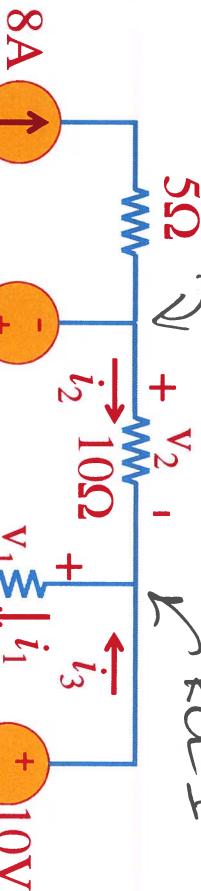
(You can submit your homework in any of the discussion sessions on either Tuesday 4/24 or Thursday 4/26)

Problem 1: (KCL, KVL, Ohm's Law) Find currents i_1 , i_2 , i_3 , and i_4 (10pts.)

KCL 2

and
KCL 1

$$V_1 = 10V$$



$$i_1 = \frac{V_1}{5} = 10/5 \Rightarrow i_1 = 2A$$

Applying KVL @ middle loop,

$$+20 + V_2 + V_1 = 0 \Rightarrow +20 + V_2 + 10 = 0$$

$$\Rightarrow V_2 = -30V$$

$$i_2 = \frac{V_2}{10} \Rightarrow i_2 = -3A$$

Applying KCL at top right node,

$$i_2 + i_3 = i_1 \Rightarrow i_3 = i_1 - i_2 = 2 + 3$$

$$\Rightarrow i_3 = 5A$$

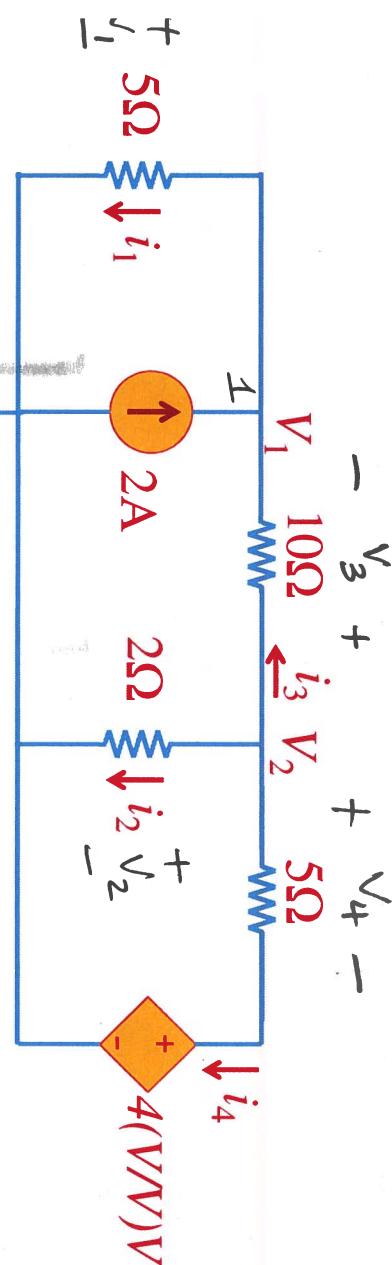
KCL at middle left node,

$$i_3 = i_2 + i_4 \Rightarrow i_4 = 8 - i_2 = 8 + 3$$

$$\Rightarrow i_4 = 11A$$

V_3 , and V_4

Problem 2: Use nodal analysis and find V_1 and V_2 and i_1 through i_4 (10pts.)



$$\text{Multiply } \textcircled{1} \text{ by } 3, \quad 9V_1 - 3V_2 = 60 \quad \textcircled{*}$$

$$\textcircled{2} + \textcircled{*} \Rightarrow +5V_2 = 60$$

$$\Rightarrow V_2 = 12V$$

$$\text{From } \textcircled{1}, \quad V_1 = \frac{20 + V_2}{3}$$

$$\Rightarrow V_1 = 10.67V$$

$$@ \text{ Node } 1, \quad \frac{V_1 - 0}{5} - 2 + \frac{V_1 - V_2}{10} = 0$$

Multiply by 10,

$$2V_1 - 20 + V_1 - V_2 = 0 \Rightarrow 3V_1 - V_2 = 20 \quad \textcircled{1}$$

$$@ \text{ node } 2, \quad \frac{V_2 - V_1}{10} + \frac{V_2 - 0}{2} + \frac{V_2 - 4V_1}{5} = 0$$

Multiply by 10,

$$\frac{V_2 - V_1}{2} + 5V_2 + 2V_2 - 8V_1 = 0$$

$$-9V_1 + 8V_2 = 0 \quad \textcircled{2}$$

$$\Rightarrow \boxed{i_3 = \frac{V_3}{10} \Rightarrow i_3 = 0.13A}$$

$$\boxed{\frac{V_4 - V_1}{5} = \frac{V_2 - 4V_1}{10} = \frac{-30.68V}{5} = -6.14A}$$

Problem 3: Use nodal analysis and find V_1 and V_2 and i_1 through i_3 (10pts.)

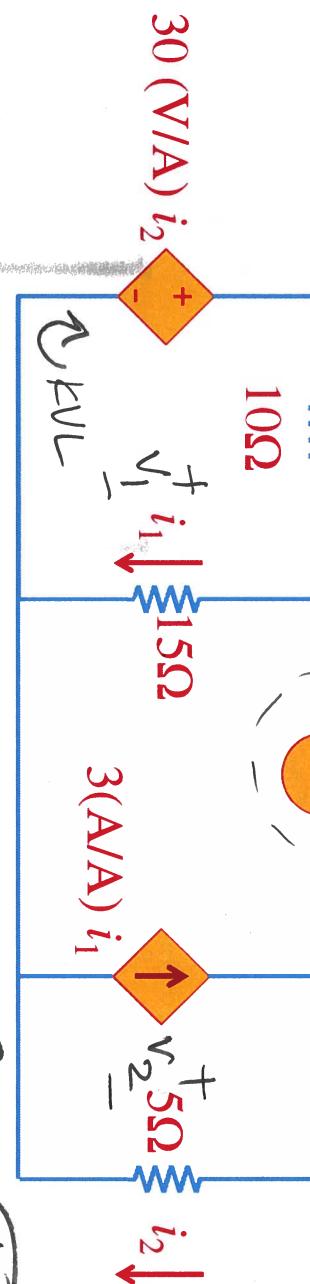
$$-\sqrt{3} - 12\sqrt{2} = 0$$

$$V_1 + 12\sqrt{2} = 0$$

i_3

V_1

V_2



① supernode,

$$\frac{V_1 - 30i_2}{10} + \frac{V_1}{15} - 3i_1 + \frac{V_2}{5} = 0$$

$$V_2 = V_1 + 3$$

$$-V_1 + V_2 = 3$$

② supernode,

$$13V_2 = 3 \Rightarrow V_2 = 3/13V$$

$$\text{from } \star, V_1 = -12V_2$$

$$\Rightarrow V_1 = -36/13V$$

$$i_1 = \frac{V_1}{15} \Rightarrow i_1 = -12/65A$$

$$i_2 = \frac{V_2}{5} \Rightarrow i_2 = 3/65A$$

$$\text{KVL @ left loop, } -30i_2 - V_3 + V_1 = 0$$

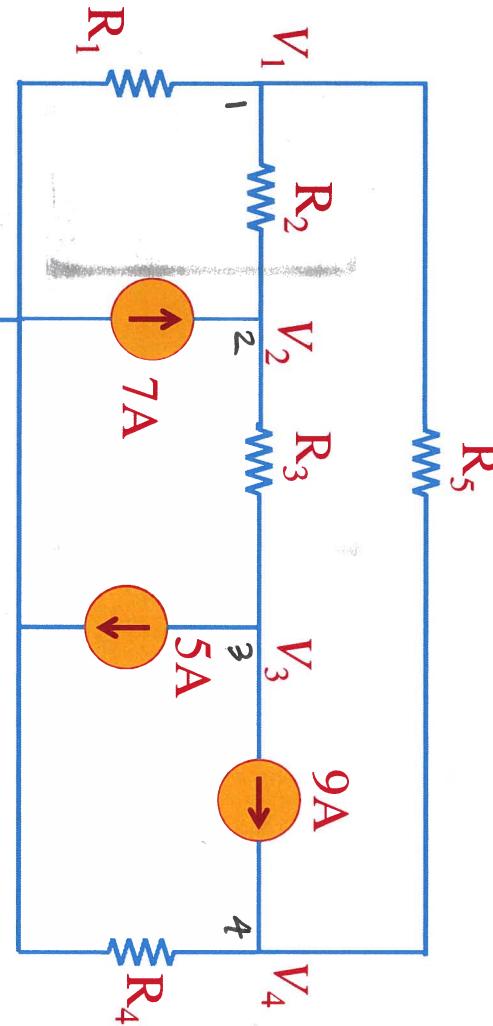
$$\Rightarrow V_3 = V_1 - 30i_2 = -36/13 - 30 \cdot 3/65$$

$$\Rightarrow V_3 = -54/130V$$

and $\sqrt{3}$

KCL @ supernode,
 $\frac{V_1 - 30i_2}{10} + \frac{V_1}{15} - 3i_1 + \frac{V_2}{5} = 0$
 $i_1 = \frac{V_1}{15}$ and $i_2 = \frac{V_2}{5}$
 put i_1 and i_2 in ①,
 $\frac{V_1 - 6V_2}{10} + \frac{V_1}{15} - \frac{V_1}{5} + \frac{V_2}{5} = 0$
 multiply by 30
 $3V_1 - 18V_2 + 2V_1 - 6V_1 + 6V_2 = 0$

Problem 4: Write all nodal voltage equations and put them in the matrix form (you do not need to solve for voltages V_1 through V_4) (10pts.)



$$@ \text{node } 4, \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_4}{R_5} = 0$$

$$@ \text{node } 2, \frac{V_2 - V_1}{R_2} - 7 + \frac{V_2 - V_3}{R_3} = 0$$

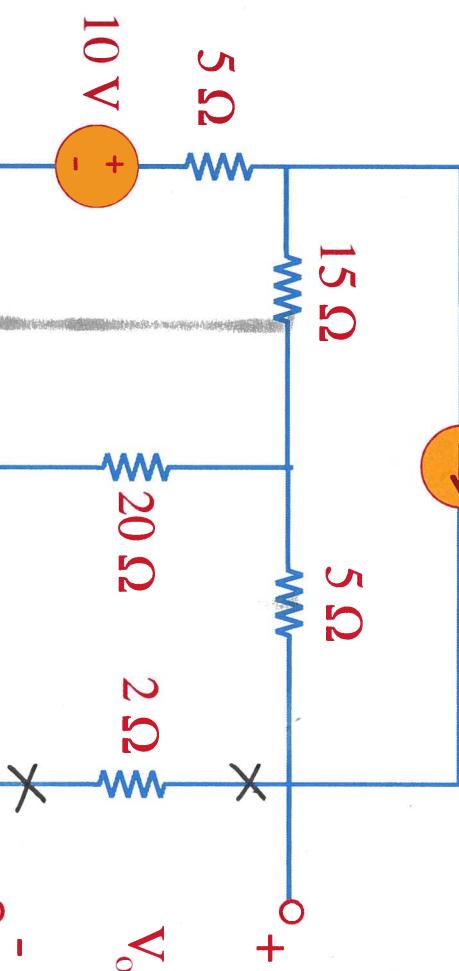
$$@ \text{node } 3, \frac{V_3 - V_2}{R_3} + 5 + 9 = 0$$

$$\therefore \text{node } 4, -9 + \frac{V_4 - V_1}{R_4} + \frac{V_4 - V_1}{R_5} = 0$$

$$\begin{bmatrix} 0 & \left(\frac{-1}{R_2}\right) & \left(\frac{1}{R_3}\right) & 0 \\ \left(\frac{-1}{R_2}\right) & 0 & \left(\frac{1}{R_2} + \frac{1}{R_3}\right) & \left(\frac{-1}{R_3}\right) \\ 0 & \left(\frac{1}{R_3}\right) & 0 & \left(\frac{-1}{R_5}\right) \\ \left(\frac{-1}{R_5}\right) & 0 & \left(\frac{1}{R_5}\right) & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \\ 0 \\ 9 \end{bmatrix}$$

Problem 5: Find V_o using Thévenin theorem (10pts.)

1 A



Using nodal analysis,

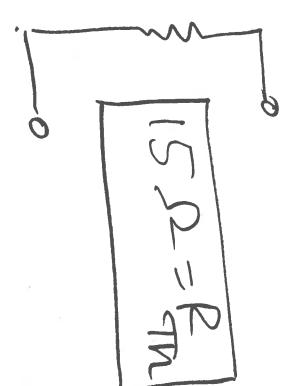
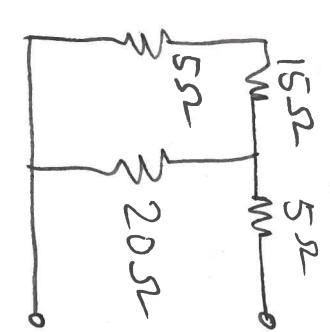
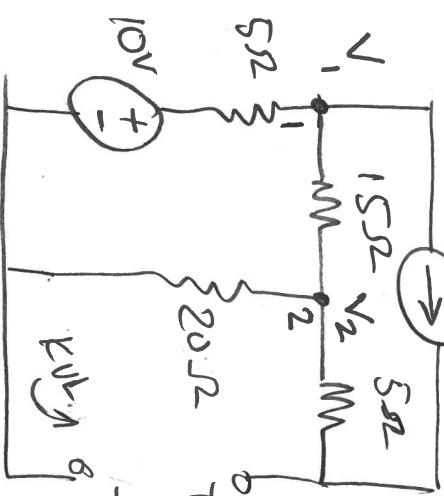
@node 1

$$\frac{V_1 - 10}{5} + \frac{V_1 - V_2}{15} + 1 = 0$$

Multiply by 15,

$$3V_1 - 30 + V_1 - V_2 + 15 = 0$$

$$+ 4V_1 - V_2 = +15 \quad \text{---(1)}$$



@ node 2,

$$\frac{V_2 - V_1}{15} + \frac{V_2}{20} - 1 = 0$$

Multiply by 60,

$$4V_2 - 4V_1 + 3V_2 - 60 = 0$$

$$-4V_1 + 7V_2 = 60 \quad \text{---(2)}$$

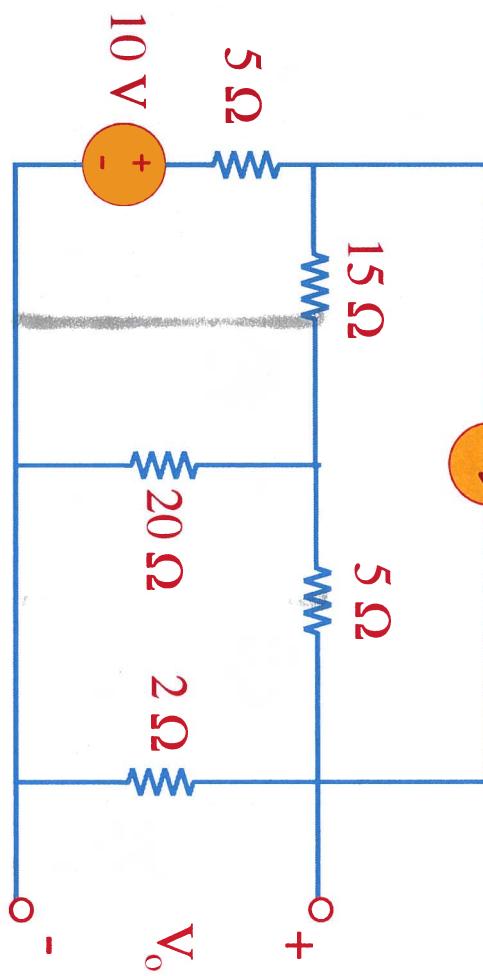
$$\textcircled{1} + \textcircled{2} \\ 6V_2 = 75 \Rightarrow V_2 = 12.5V$$

$$\text{Using KVL, } -V_{oc} + 5 + 12.5 = 0$$

$$\Rightarrow V_{oc} = 17.5V$$

Problem 5: Find V_o using Thévenin theorem (10pts.)

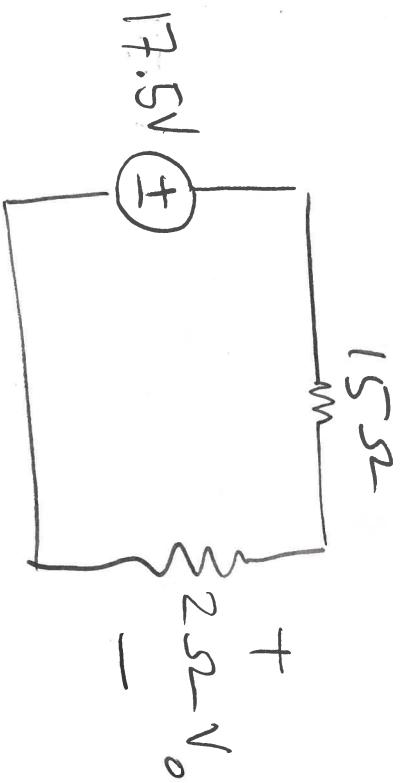
1 A



Using voltage divider,

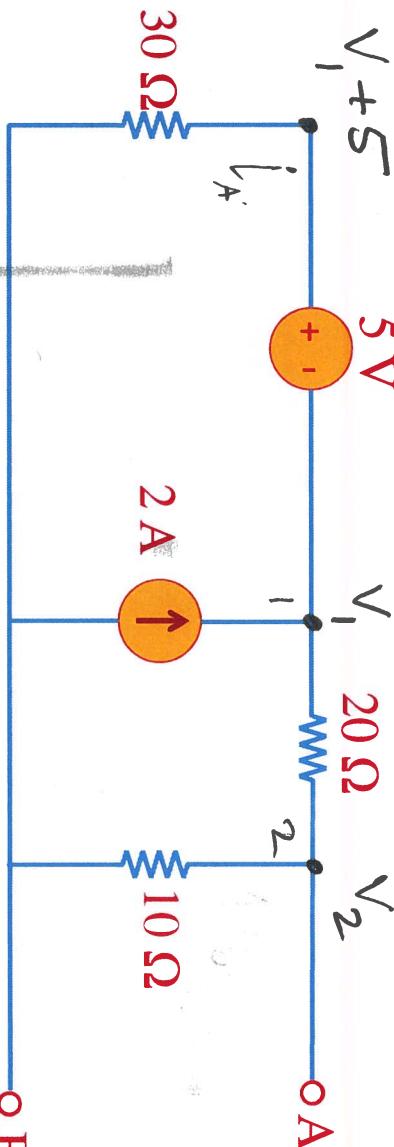
$$V_o = 17.5 \frac{2}{15 + 2}$$

$$\Rightarrow V_o = 2.06 \text{ V}$$



Problem 6: Find both the Thevenin and Norton equivalent circuits at terminals AB.

(10pts.)



A circuit diagram with four nodes labeled 1, 2, 3, and A. Node 1 is at the top, node 2 is below it, node 3 is at the bottom, and node A is at the far right. A vertical blue line connects nodes 1 and 2. A horizontal blue line connects nodes 2 and 3. A horizontal blue line connects node 3 and node A. At node 1, there is a voltage source $V_1 + 5$ connected to ground. At node 2, there is a voltage source V_2 connected to ground. At node 3, there is a voltage source $5V$ connected to ground. A red circle at node 1 contains a plus sign above a minus sign, indicating the positive terminal of the $V_1 + 5$ source is at node 1. A red circle at node 2 contains a minus sign above a plus sign, indicating the negative terminal of the V_2 source is at node 2. A red circle at node 3 contains a plus sign above a minus sign, indicating the positive terminal of the $5V$ source is at node 3. A blue spring symbol is positioned between nodes 2 and 3. A dependent current source, indicated by a circle with a dot and a cross, is located between nodes 1 and 2. The text "20 Ω" is written next to the spring symbol.

$$\frac{V_2 - V_1}{20} + \frac{V_2}{10} = 0$$

Multiply by 20,

$$V_2 - V_1 + 2V_2 = 0$$

$$\begin{aligned} & \text{Multiply by } 20, \\ & V_2 - V_1 + 2V_2 = 0 \\ & -V_1 + 3V_2 = 0 \quad \text{--- (2)} \end{aligned}$$

Multiply ② by 5 and add to ①

$$\Rightarrow \frac{z}{\sqrt{\pi n}} = 55/60 \text{ V}$$

To Find P_{ik} , use

$$\begin{aligned}
 R_{TH} &= 50 \Omega \\
 \frac{R_{TH}}{R_N} &= 25/3 \quad \Rightarrow R_N = 12 \Omega \\
 \frac{1}{R_N} &= \frac{1}{12} \Omega^{-1} \\
 \frac{1}{R_N} &= \frac{1}{12} \Omega^{-1}
 \end{aligned}$$

$$\frac{V_1 + 3}{30} - 2 + \frac{\frac{V_1 - V_2}{20}}{20} = 0$$

Multiply by 60,

$$\frac{2V_1 + 10 - 120 + 3V_1 - 3V_2}{60} = 0$$

$$5V_1 - 3V_2 = 110 \quad \textcircled{1}$$

Applying nodal analysis @ node 1,

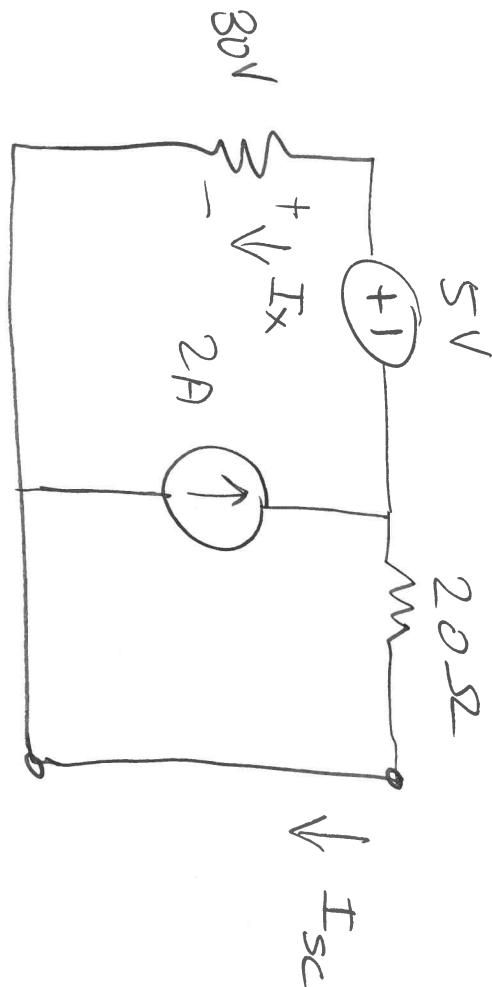
$$i_A = \frac{V+5}{30}$$

$$\frac{V_1 + 5}{30} - 2 + \frac{V_1 - V_2}{20} = 0$$

$$2V_1 + 10 - 120 + \underline{3V_1} - 3V_2 = 0$$

$$\frac{5V_1 - 3V_2}{1} = 110 \quad \textcircled{1}$$

Problem 6. A 1T. method to find K_{TH}



$$-30I_x + 5 + 20I_{sc} = 0$$

$$\Rightarrow 4I_{sc} - 6I_x = -1 \quad \text{--- (1)}$$

$$2 = I_x + I_{sc} \Rightarrow I_x = 2 - I_{sc} \quad \text{--- (2)}$$

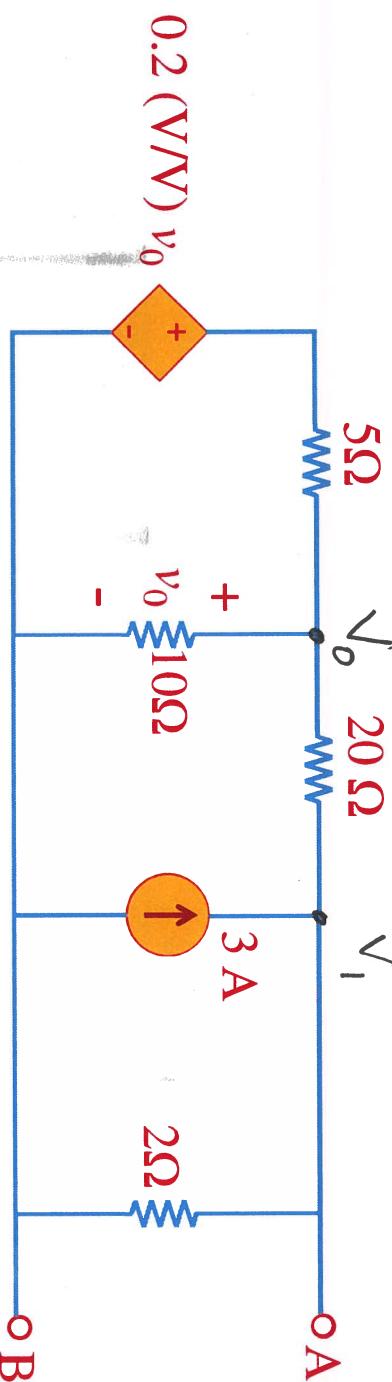
② in ①

$$\frac{4I_{sc} - 6(2 - I_{sc})}{10I_{sc}} = -1 \Rightarrow I_{sc} = 11/10 \text{ A}$$

$$\boxed{R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{\frac{5+30}{10}}{\frac{11}{10}} = 25/3 \Omega = R_N}$$

Problem 7: Find both the Thevenin and Norton equivalent circuits at terminals AB.

(10pts.)



Nodal analysis @ V_o ,

$$\frac{V_o - 0 - 2V_o}{5} + \frac{V_o}{10} + \frac{V_o - V_i}{20} = 0$$

$$0.3V_o - 0.05V_i = 0 \quad \textcircled{1}$$

$$\textcircled{2} \quad V_i'$$

$$\frac{V_i - V_o}{20} - 3 + \frac{V_i}{2} = 0$$

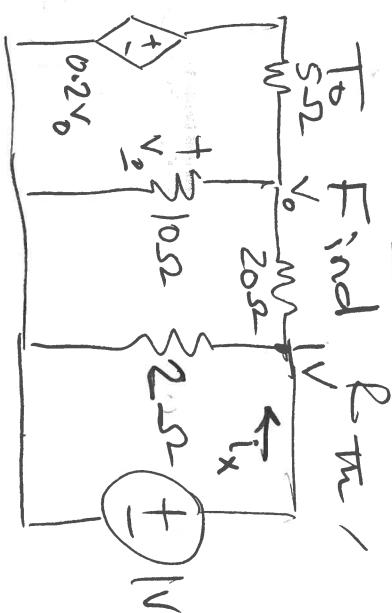
$$-0.05V_o + 0.55V_i = 3 \quad \textcircled{2}$$

Multiply $\textcircled{1}$ by 11

$$3.4V_o - 0.55V_i = 0 \quad \textcircled{4}$$

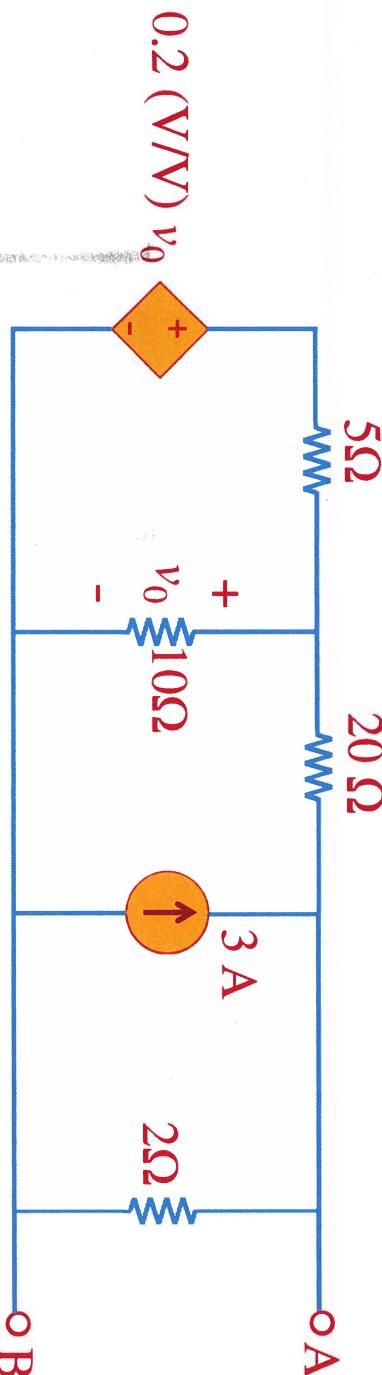
$$\textcircled{2} + \textcircled{4} - 3.36V_o = 3 \Rightarrow V_o = 0.89V$$

From $\textcircled{1}$, $V_i = 5.52V = V_{oc} = V_{th}$



Problem 7: Find both the Thevenin and Norton equivalent circuits at terminals AB.

(10pts.)



Nodal analysis @ v_o

$$\frac{v_o - 0.2v_o}{5} + \frac{v_o}{10} + \frac{v_o - 1}{20} = 0$$

Multiply by 20

$$4v_o - 0.8v_o + 2v_o + v_o - 1 = 0$$

$$v_o = 0.16 \text{ V}$$

@ IV node,

$$\frac{1 - v_o}{20} + \frac{1}{2} - i_x = 0$$

$$\Rightarrow i_x = 0.542 \text{ A}$$

$$R_{TH} = \frac{1 \text{ V}}{0.542} = 1.85 \Omega = R_N$$

$$I_{SC} = \frac{\sqrt{V_L}}{R_N} = 2.93 \text{ A}$$