EECS/CSE 70A Network Analysis I

Homework #5

Due on or before
5/22/2018, Tuesday at 5:00PM

(You can submit homework in either of the discussion sessions only on Tuesday 5/22 or put it in the box near EH 4404 on 5/22 by 5:00PM)

Problem 1: (10 pts)

$$u = \frac{A + jB}{C + jD} \times \frac{C - jD}{C - jD} = \frac{(AC+BD) + j(BC-AD)}{C^2 + D^2}$$

A, B, C, and D are real.

$$= \frac{AC+BD}{C^2+D^2} + j\,\frac{BC-AD}{C^2+D^2}$$

$$\begin{array}{r} A + jB \\ C - jD \\ \hline AC + jBC \\ -jAD + BD \\ \hline (AC+BD) + j(BC-AD) \end{array}$$

$$\frac{\begin{array}{c} C + jD \\ C - jD \end{array}}{C^2 + D^2}$$

2 a) Find Re(u) $= \frac{AC+BD}{C^2+D^2}$

2 b) Find Im(u) $= \frac{BC-AD}{C^2+D^2}$

2 c) Express u as (X + jY)

2 d) Express u as ($r\,e^{j\theta}$) $r = \sqrt{\left(\frac{AC+BD}{C^2+D^2}\right)^2 + \left(\frac{BC-AD}{C^2+D^2}\right)^2}$, $\theta = \tan^{-1}\left(\frac{BC-AD}{AC+BD}\right)$

2 e) Find Re($u\,e^{j\omega t}$) $= Re\{r\,e^{j(\omega t+\theta)}\}$

$= Re\{r\cos(\omega t+\theta) + j\,r\sin(\omega t+\theta)\}$

$= r\cos(\omega t+\theta)$

Problem 2a: (10 pts)

Given $v(t) = 10\cos(\omega t - \pi/4)$ volts. Find the phasor **V** that represents $v(t)$. Express **V** as both $x+jy$ and $re^{j\theta}$.

$$\boxed{5}$$

$$\bar{V} = 10 \angle -\pi/4 = 10 e^{-j\pi/4} \text{ V}$$

$$x = r\cos\theta = 10\cos(-\pi/4) = 5\sqrt{2}$$

$$y = r\sin\theta = 10\sin(-\pi/4) = -5\sqrt{2}$$

$$\bar{V} = +5\sqrt{2} - j5\sqrt{2} \text{ V}$$

$$\boxed{5}$$

Problem 2b: (10 pts)

Given i(t) = 2sin(5t+π/6) amps. Find the phasor **I** that represents i(t). Express **I** as both x+jy and $re^{j\theta}$.

$$i(t) = 2\cos(5t + \pi/6 - \pi/2) = 2\cos(5t - \pi/3)\ A$$

$$\bar{I} = 2\angle{-\pi/3} = 2e^{-j\pi/3}\ A \quad \boxed{5}$$

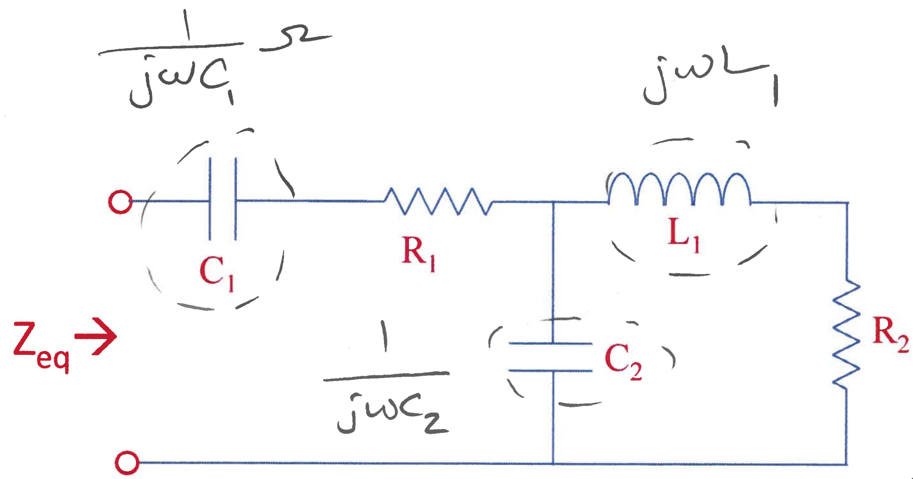
$$x = r\cos\theta = 2\cos(-\pi/3) = 1$$

$$y = r\sin\theta = 2\sin(-\pi/3) = -\sqrt{3}$$

$\boxed{5}$ $$\bar{I} = 1 - j\sqrt{3}\ A$$

Problem 3a: (10 pts)

Find the impedance Z_{eq} if L is the inductance, C is the capacitance, and R is the resistance. No need to simplify your answer as x+jy or $re^{j\theta}$.

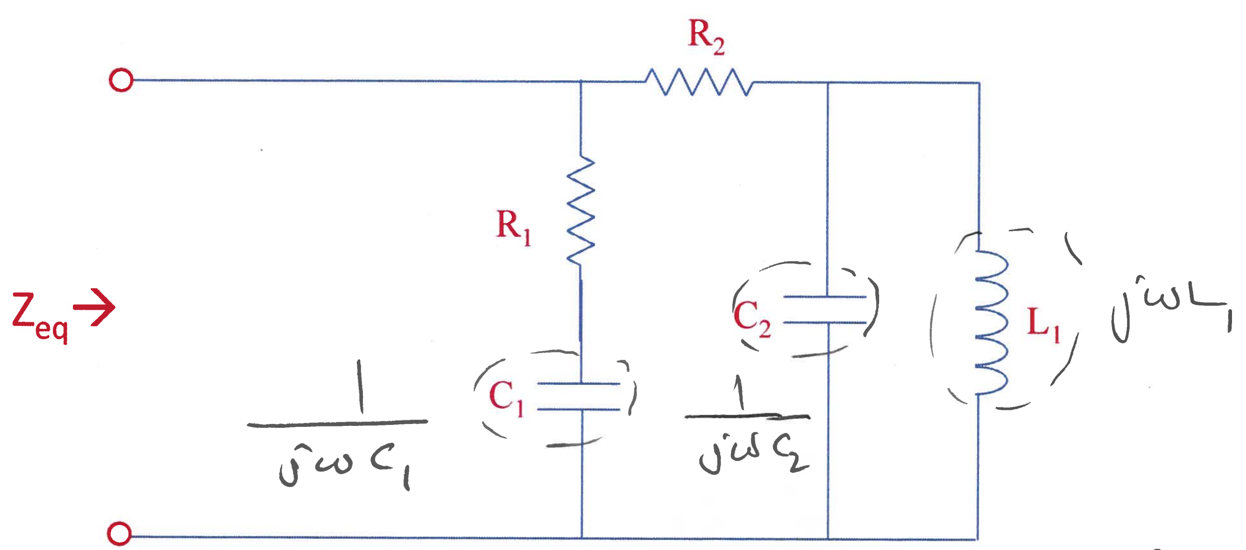


$$Z_{eq} = \left[(j\omega L_1 + R_2) \parallel \left(\frac{1}{j\omega C_2} \right) \right] + \left[R_1 + \frac{1}{j\omega C_1} \right] \quad \boxed{5}$$

$$= \left[\frac{j\omega L_1 + R_2 \cdot \frac{1}{j\omega C_2}}{j\omega L_1 + R_2 + \frac{1}{j\omega C_2}} + R_1 + \frac{1}{j\omega C_1} \right] \Omega \quad \boxed{5}$$

Problem 3b: (10 pts)

Find the impedance Z_{eq} if L is the inductance, C is the capacitance, and R is the resistance. No need to simplify your answer as x+jy or $re^{j\theta}$.



$$Z_{eq} = \left[\left(j\omega L_1 \parallel \frac{1}{j\omega C_2}\right) + R_2\right] \parallel \left[R_1 + \frac{1}{j\omega C_1}\right] \quad \boxed{5}$$

$$= \left[\underbrace{\frac{j\omega L_1 \cdot \frac{1}{j\omega C_2}}{j\omega L_1 + \frac{1}{j\omega C_2}} + R_2}_{A}\right] \parallel \left[\underbrace{R_1 + \frac{1}{j\omega C_1}}_{B}\right] = \frac{AB}{A+B} \; \Omega \quad \boxed{5}$$

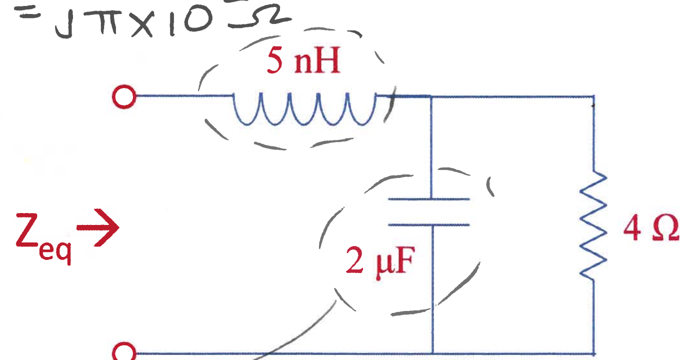
Problem 3c: (10 pts)

$\omega = 2\pi f$

Find the impedance Z_{eq} if f = 1 MHz. Express the answer as both x+jy and $re^{j\theta}$.

$j\omega L = j(2\pi \times 1 \times 10^{6} \times 5 \times 10^{-9})$

$= j\pi \times 10^{-2}\ \Omega$



$$\frac{1}{j\omega C} = \frac{1}{j(2\pi \times 10^{6}) \times 2 \times 10^{-6}}$$

$$= \frac{1}{j4\pi}\ \Omega$$

$$Z_{eq} = \left(4 \parallel \frac{1}{j4\pi}\right) + j\pi \times 10^{-2}$$

$$= \frac{\frac{1}{j\pi}}{4 + (1/j4\pi)} + j\pi \times 10^{-2}$$

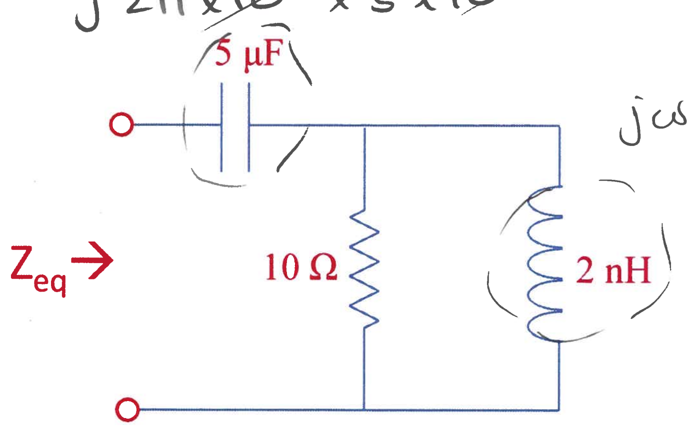
$$= 0.00158 + j0.04813\ \Omega \quad \boxed{5}$$

$$= 0.04816 \angle +88.12^{\circ}\ \Omega \quad \boxed{5}$$

Problem 3d: (10 pts)

Find the impedance Z_{eq} if f = 1 MHz. Express the answer as both x+jy and $re^{j\theta}$.

$$\frac{1}{j\omega C} = \frac{1}{j\,2\pi \times \cancel{10^{6}} \times 5 \times \cancel{10^{-6}}} = \frac{1}{j\,10\,\pi}\ \Omega$$



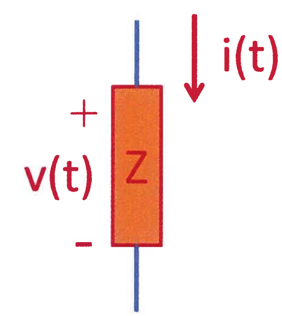
$$j\omega L = j\,2\pi \times 10^{6} \times 2 \times 10^{-9}$$
$$= j\,4\pi \times 10^{-3}\ \Omega$$

$$Z_{eq} = \left(10 \parallel j4\pi \times 10^{-3}\right) + \frac{1}{j\,10\pi} = \frac{j4\pi \times 10^{-2}}{10 + j4\pi \times 10^{-3}} + \frac{1}{j\,10\pi}$$

$$= 0.000016 - j0.019\ \Omega \quad \boxed{5}$$

$$= 0.019 \angle -89.95^{\circ}\ \Omega \quad \boxed{5}$$

Problem 4a: (10 pts)



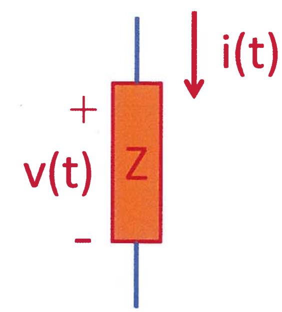
Given Z = 3∠10° ohms. Find i(t) if v(t) = 8cos(2t+π/4) volts.

$$\overline{V} = 8\angle \pi/4$$

$$\overline{I} = \frac{\overline{V}}{\overline{Z}} = \frac{8\angle 45^\circ}{3\angle 10^\circ} = \frac{8}{3}\angle 35^\circ \text{ A} \quad \boxed{5}$$

$$i(t) = \frac{8}{3}\cos(2t + 35^\circ) \text{ A} \quad \boxed{5}$$

Problem 4b: (10 pts)



Given Z = 3∠10° ohms. Find v(t) if i(t) = 4cos(20t-π/3) amps.

$\bar{I} = 4 \angle \overbrace{-\pi/3}^{-60^\circ} \ A$ $\boxed{5}$

$\bar{V} = \bar{I}\bar{Z} = (4 \angle -60^\circ)(3 \angle 10^\circ) = 12 \angle -50^\circ \ V$

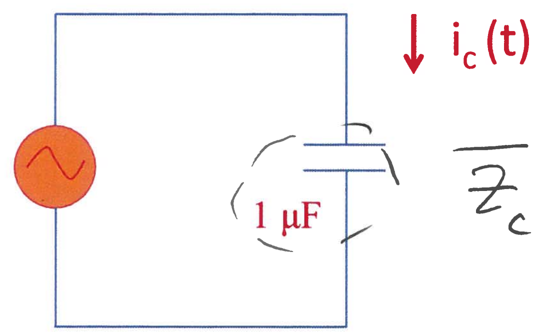
$v(t) = 12 \cos(20t - 50^\circ) \ V$ $\boxed{5}$

Problem 5a: (10pts)

Find $i_C(t)$. Hint: convert the voltage source into a phasor, then find the current phasor for the capacitor, then convert back to $i_C(t)$.

$\omega = 2\pi f = 40$

$V_s(t) = 100\cos(40t + 30^\circ)$ volts



$\overline{Z}_c = \dfrac{1}{j\omega C} = \dfrac{1}{j(40)1\times10^{-6}}$

$= \dfrac{1}{j4\times10^{-5}} = 25{,}000\angle{-90^\circ}\ \Omega$ $\boxed{3}$

$\overline{V}_s = 100\angle 30^\circ$

$\overline{I}_c = \dfrac{\overline{V}_s}{\overline{Z}_c} = \dfrac{100\angle 30^\circ}{25{,}000\angle{-90^\circ}}$

$= \dfrac{1}{250}\angle 120^\circ$ A $\boxed{5}$

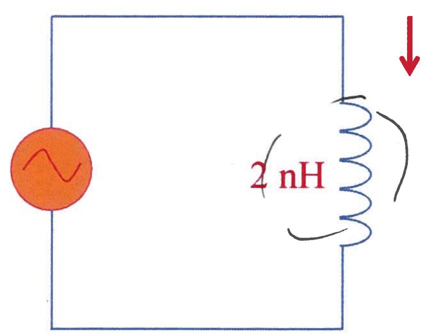
$\Rightarrow i_c(t) = \dfrac{1}{250}\cos(40t + 120^\circ)$ A $\boxed{2}$

Problem 5b: (10pts)

Find $i_L(t)$. Hint: convert the voltage source into a phasor, then find the current phasor for the inductor, then convert back to $i_L(t)$.

$\boxed{3}$

$\omega = 10$



$\downarrow i_L(t)$

$$j\omega L = j(10)\times 2\times 10^{-9}$$
$$= j\,2\times 10^{-8}\ \Omega$$
$$= 2\times 10^{-8}\angle 90^\circ\ \Omega$$

$$\underbrace{V_s(t) = 60\cos(10t + 45^\circ)\text{ volts}}$$

$$\bar{V}_s = 60\angle 45^\circ$$

$$I_L = \frac{\bar{V}_s}{\bar{Z}_L} = \frac{60\angle 45^\circ}{2\times 10^{-8}\angle 90^\circ}$$

$$= 30\times 10^{8}\angle -45^\circ\ A \quad \boxed{5}$$

$$\Rightarrow i_L(t) = 30\times 10^{8}\cos(10t - 45^\circ)\ A \quad \boxed{2}$$