EECS/CSE 70A Network Analysis I

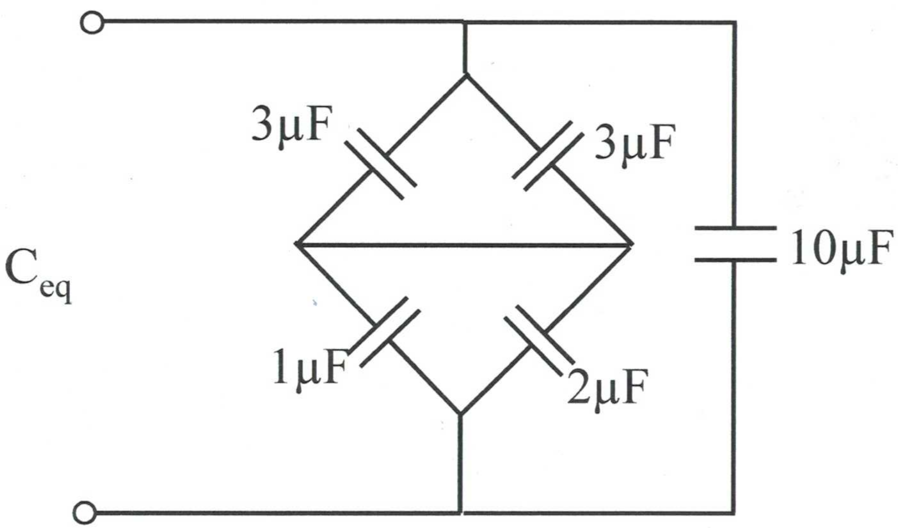
Homework #4

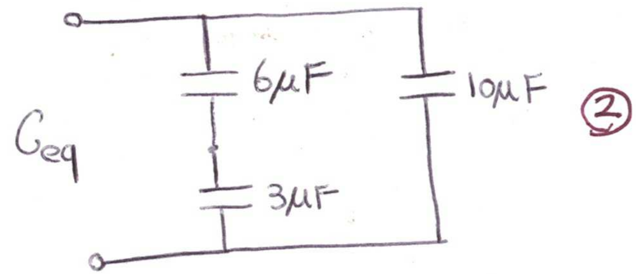
Due on or before

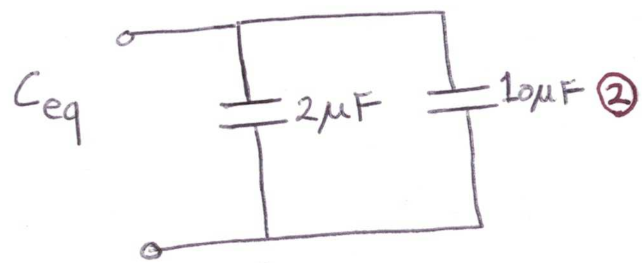
5/11/2018, Friday 5pm in the box in front of EH 4404

(You can submit your homework in any of the Tuesday or Thursday discussions before or on 5/11/2018)

Problem 1: Find C_{eq} (5pts).

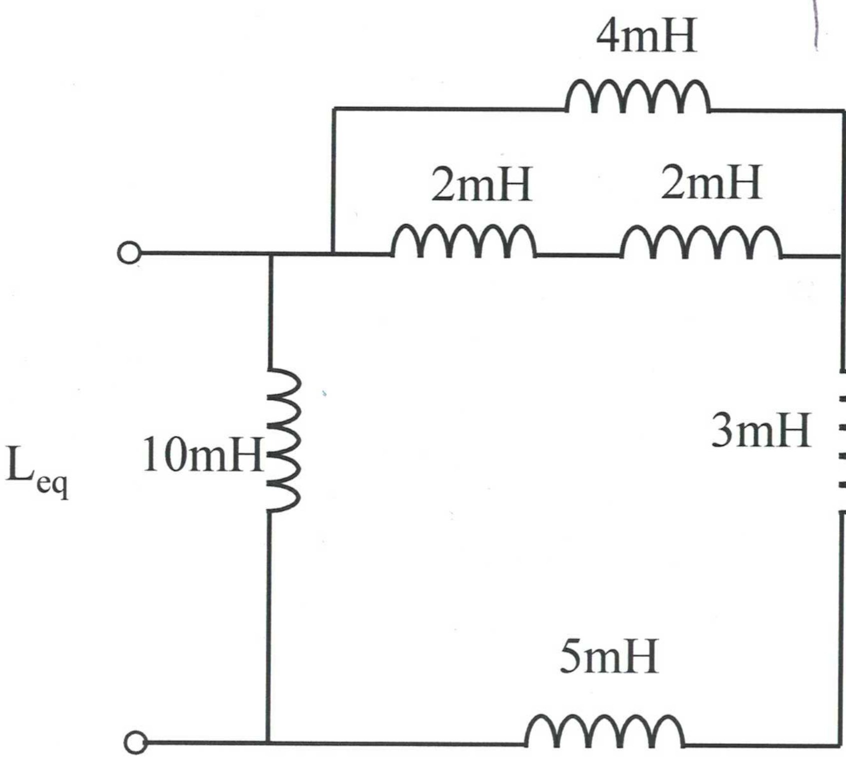


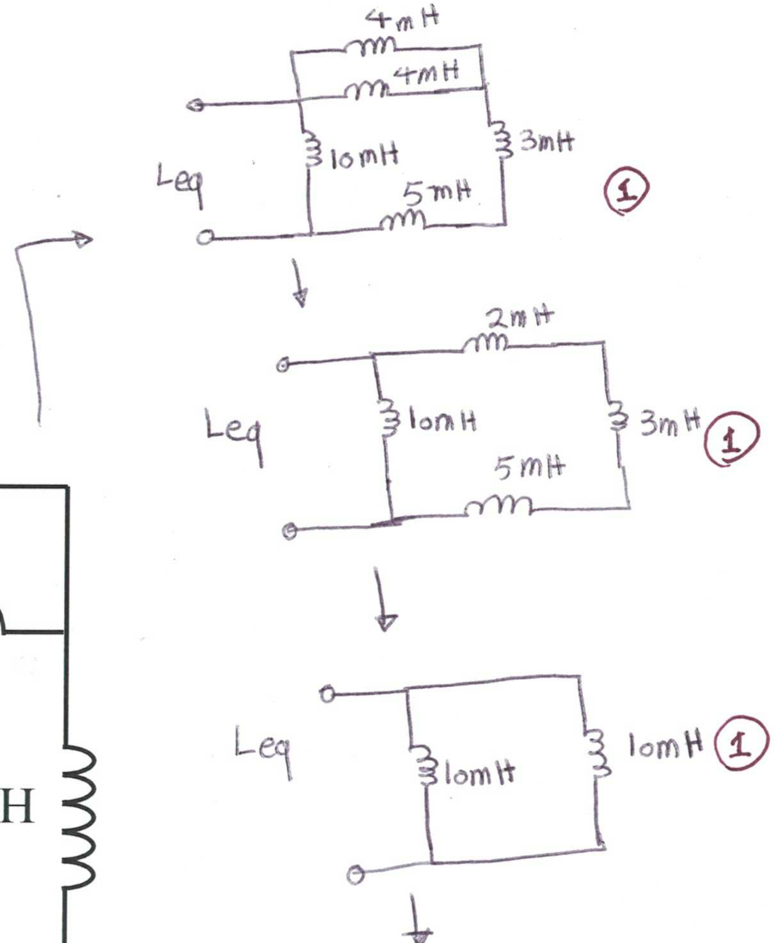




$C_{eq} = 12\mu F$ 1

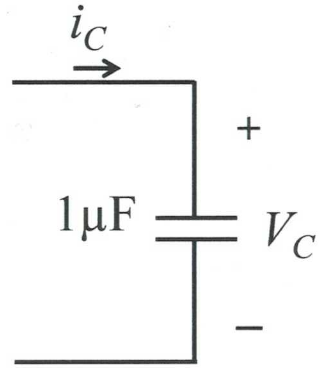
Problem 2: Find L_{eq} (5pts).





$L_{eq} = 10mH \parallel 10mH = 5mH$ 2

Problem 3: The current flowing through the capacitor is given as a function of time in the following figure.
Plot the voltage of the capacitor, $V_C(t)$, and the charge of the capacitor, $q(t)$. Assume the initial voltage of the capacitor is zero ($V_C(0)=0$).
Mark the axis of your plots with numbers and units. (10pts)



$$V_C(t) = V_C(0) + \frac{1}{C}\int_0^t i_C(t')\,dt'$$

$$V_C(t) = \begin{cases} 0 & 0 \leq t \leq 1\text{ms} \\ 10^3(t-10^{-3}) & 1\text{ms} \leq t < 3\text{ms} \\ 2V & 3\text{ms} \leq t \leq 4\text{ms} \\ 2V + 10^3(t-4\times10^{-3}) & 4\text{ms} \leq t < 5\text{ms} \\ 3V & t \geq 5\text{ms} \end{cases}$$

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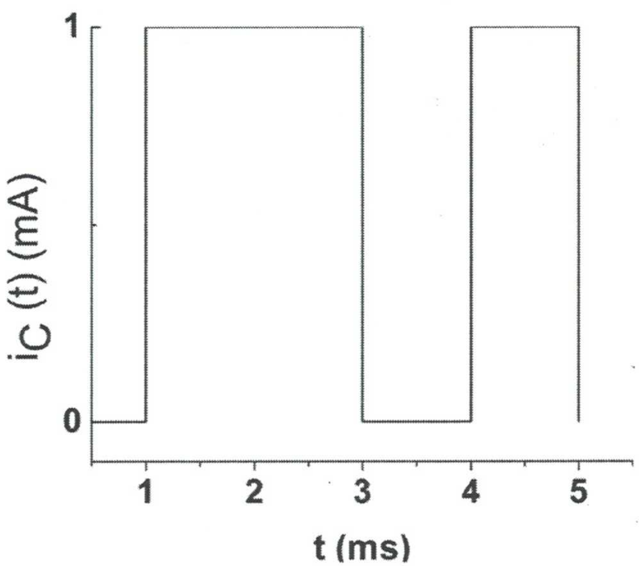
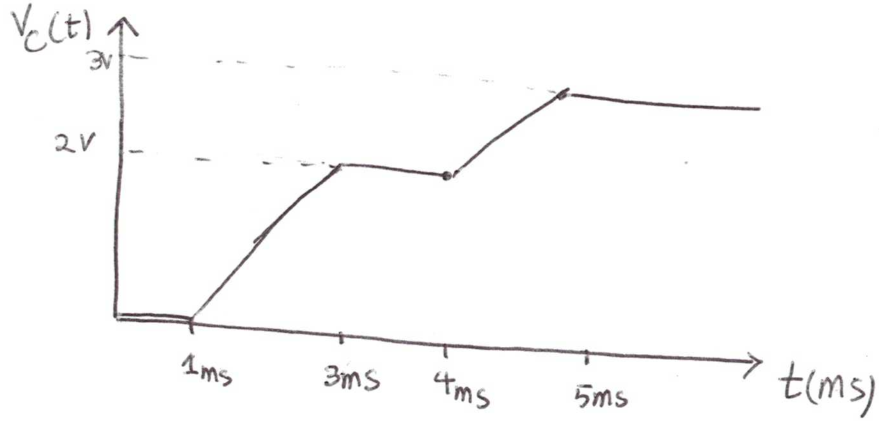
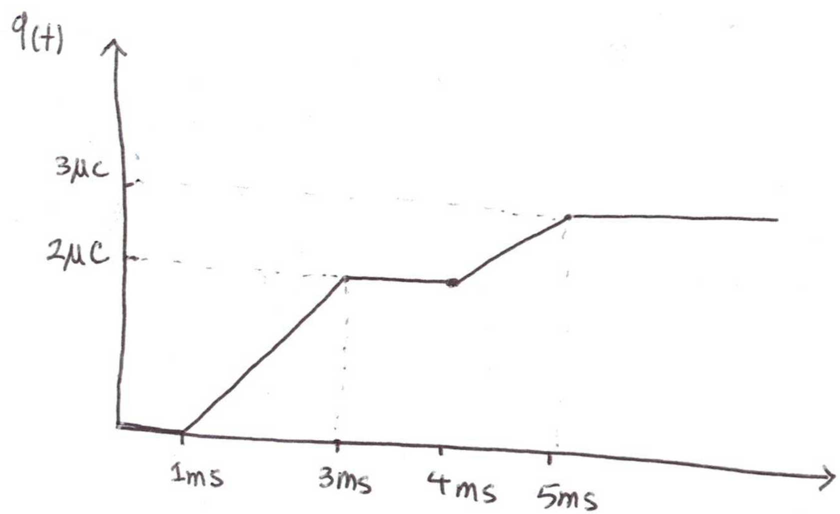




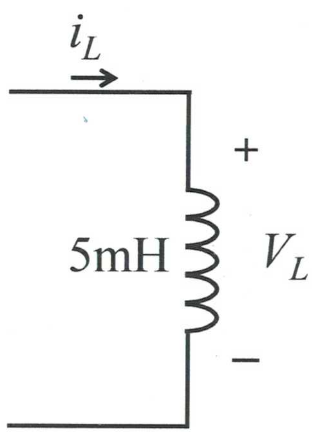
②

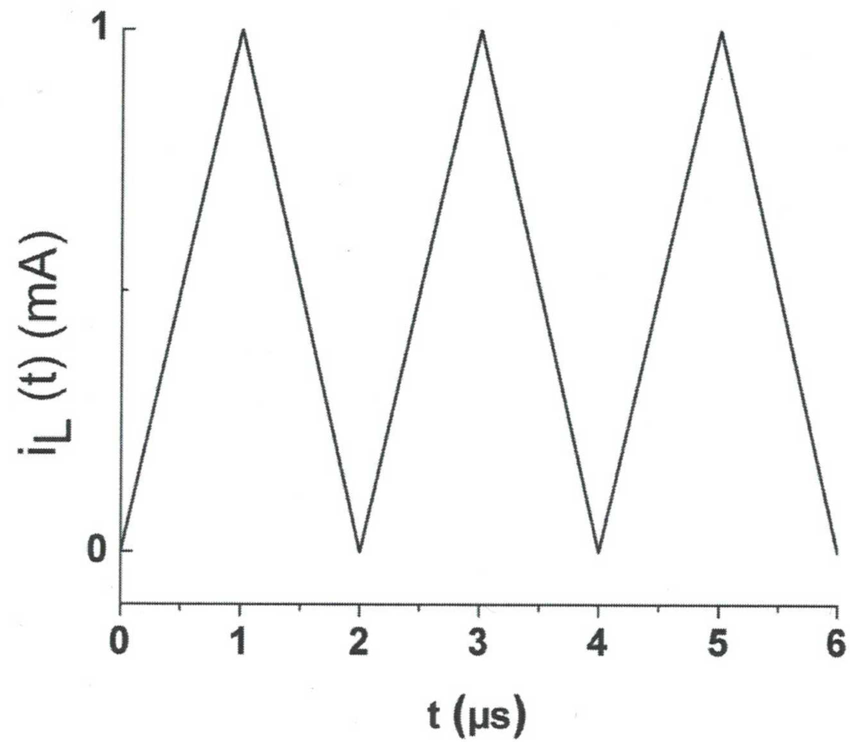
$q = C \cdot V_c \Rightarrow q(t) = 10^{-6} V_c(t)$ ①



②

Problem 4: The current flowing through the in the inductor is given as a function of time in the following figure. Plot the voltage of the inductor, $V_L(t)$.
Mark the axis of your plot with numbers and units. (5pts)



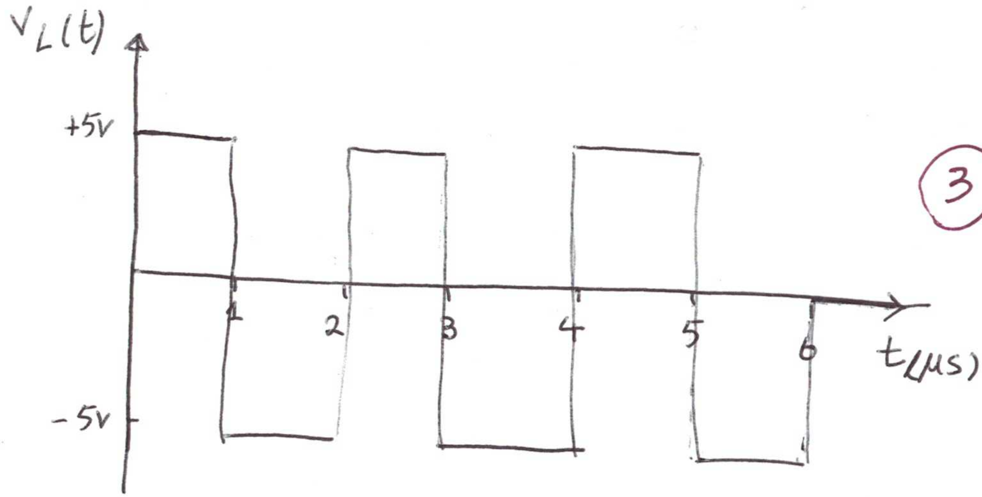


$$V_L(t) = L \cdot \frac{di_L(t)}{dt}$$

for $0 \leqslant t \leqslant 1\mu s$ $\quad V_L(t) = 5\times10^{-3}\ \frac{1\times10^{-3} - 0}{1\times10^{-6}} = 5V$ 1

for $1\mu s \leqslant t < 2\mu s$ $\quad V_L(t) = 5\times10^{-3} \times \frac{0 - 1\times10^{-3}}{1\times10^{-6}} = -5V$ 1

and then this repeats up to $t = 6\mu s$. So we have:



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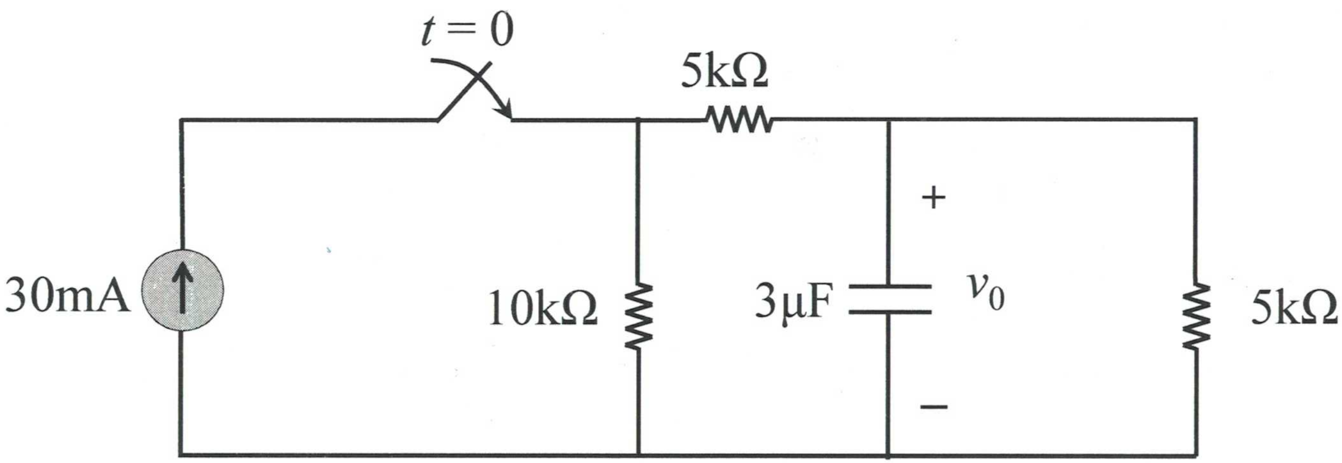
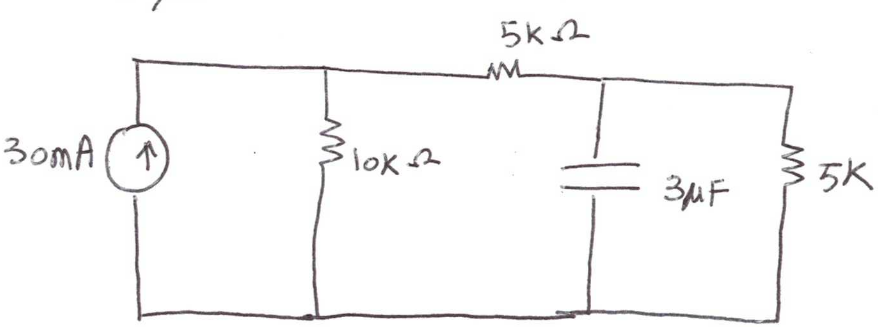
Problem 5: (RC circuit) In the circuit below the switch cloases at t=0. Write the expression for the voltage v_0 for t>0. Please clearly show the time constant calculation, initial and steady state voltage across the 3μF capacitor (35pts.)

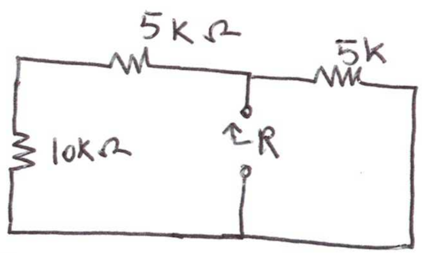


befor t<o $V_c\,t=0 \Rightarrow \boxed{V_o(0)=0}$ 10

t>o

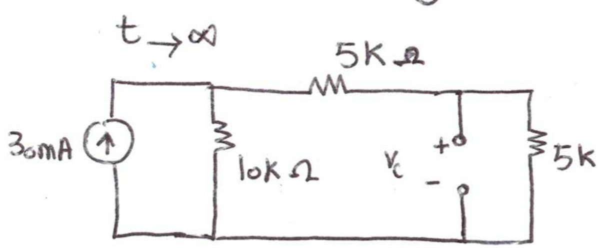


to find time constant $\tau = RC$, we need to find the Resistance seen from the terminals of capacitor while canceling independent voltage and current sources. (make independ voltage source wire, make independent current soure open)



$R = 15K \parallel 5K = 3.75 K\Omega \Rightarrow \boxed{\tau = RC = 11.25 \text{ ms}}$ 5 5

To find $V_C(\infty)$ (steady state value), we replace Capacitor with open circuit

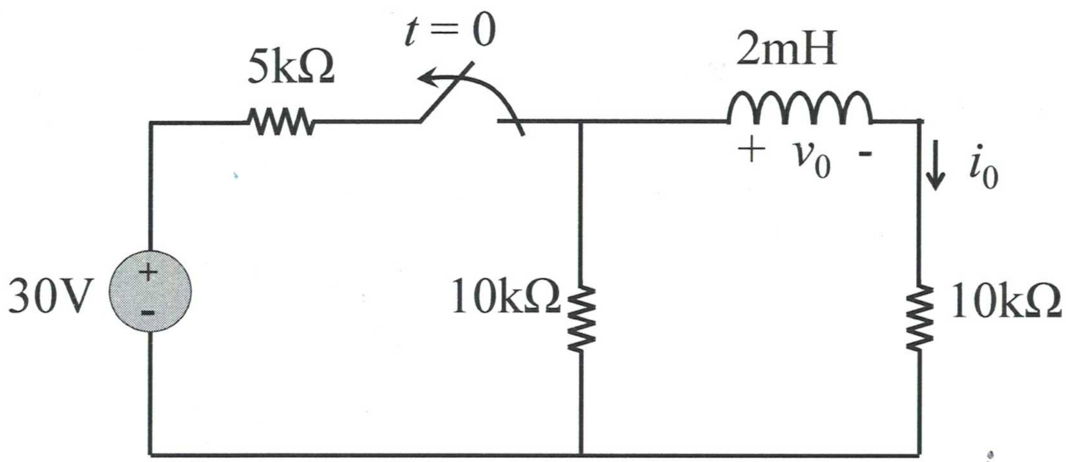


$\boxed{V_C(\infty) = 30mA \times \frac{1}{2} \times 5K\Omega = 75V}$ 10

Finally we have $V_C(t) = V_C(\infty) + (V_C(0) - V_C(\infty))\, e^{-\frac{t}{\tau}} \Rightarrow$

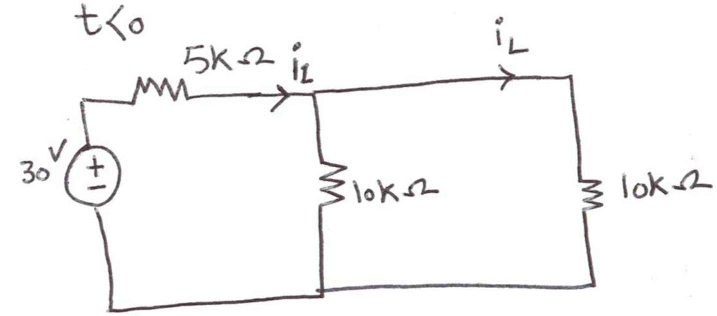
$$\boxed{V_C(t) = 75\left(1 - e^{-\frac{t}{11.25 \times 10^{-3}}}\right) = V_o(t)}$$ 5

Problem 6: (RL circuit) In the circuit below the switch opens at t=0. Write the expressions <u>first</u> for the current i_0 and <u>then</u> the voltage v_0 for t>0. Please clearly show the time constant calculation, initial and steady state current through the inductor. (40pts.)



@ t<0 the switch was closed and the inductor has reached steady state (so it acts like a short circuit)

t<0



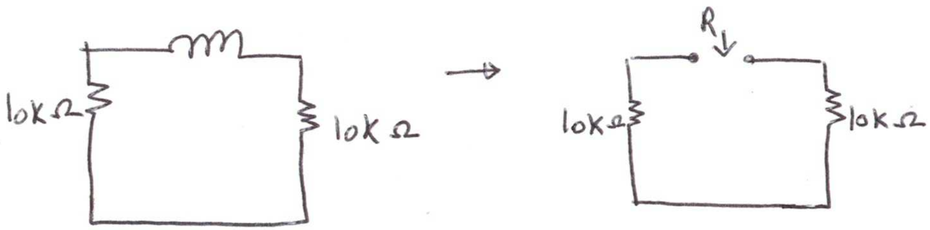
$$i_1 = \frac{30V}{(5k\Omega + 10k \parallel 10k)} = 3mA$$

Current division formula $i_L = \frac{10k}{10k + 10k} \cdot i_1 = 1.5mA$

so we have $\boxed{i_L(0) = 1.5mA}$ 10

To find time constant we need to find the resistance seen from the inductor while all independent sources are canceled.

$t > 0$

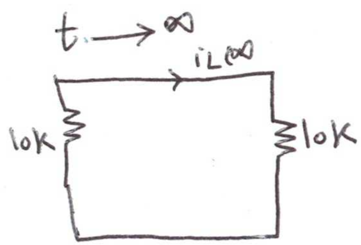


$R = 10k\Omega + 10k\Omega = 20k\Omega$

$$\boxed{\tau = \frac{L}{R} = \frac{2mH}{20k\Omega} = 10^{-7}\,s = 0.1\,\mu s}$$ 10

for steady state value ($i_L(\infty)$), we replace the inductor with wire or short circuit

$t \to \infty$



since there is no source in the circuit, all the values are zero @ $t \to \infty$. $\boxed{i_L(\infty) = 0}$ 5

Finally $i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-\frac{t}{\tau}} \rightarrow \boxed{i_L(t) = 1.5\, e^{-\frac{t}{10^{-7}}}\ [mA]}$ 10

$$v_L(t) = L\frac{di_L}{dt} = 1.5 \times 10^{-3} \times \left(-\frac{1}{10^{-7}}\right) e^{-\frac{t}{10^{-7}}} \times 2 \times 10^{-3} \Rightarrow \boxed{V_o = v_L(t) = -30\, e^{-\frac{t}{10^{-7}}}\ [V]}$$ 5