

EXPERIMENT 6:

TIME AND FREQUENCY DOMAIN REPRESENTATION OF SIGNALS

The objectives of this experiment are to generate and modulate signals and measure them both in time and frequency domains.

I. BACKGROUND

I.1 Frequency Analysis

Signals can be detected from our environment or other sources; for example, phone calls and FM radio signals are among the most common signals we use in our daily life. The structure of these real-life signals is out of framework of this course. In this lab session, we are going to see some useful instruments which are widely used in circuits and electronics.

Dynamical behaviors in time domain is usually not sufficient to fully analyze the signals. Frequency domain representation of signals can give us idea about the signal quality. In this experiment, we first generate periodic signals as a function of time and then analyze them both in time and frequency domains. Since the generated signals are in time domain, their time domain measurement can be directly obtained using *oscilloscope*. The signals are transformed and visualized into the frequency domain on the *spectrum analyzer*. Transformation of signals from time to frequency domain can be analyzed by using Fourier series and *Fourier transformation* as explained below.

Any periodic signal can be represented by infinite series of sine and cosine functions as:

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)) \quad (1)$$

where,

$$\cos(n\omega_0 t) = \frac{1}{2} (e^{jn\omega_0 t} + e^{-jn\omega_0 t}) \quad (2.a)$$

$$\sin(n\omega_0 t) = \frac{1}{2j} (e^{jn\omega_0 t} - e^{-jn\omega_0 t}) \quad (2.b)$$

By using (2.a) and (2.b), (1) can be rewritten as:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \quad (3)$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt \quad (4)$$

where $\omega_0 = 2\pi / T$. T and ω_0 are period and fundamental frequency, respectively, of the signal; and c_n is known as Fourier coefficient. Multiplying both sides of (3) with $e^{-j\omega t}$ and then integrating both sides of resulting equation with $\int_{-\infty}^{\infty} dt$, (3) becomes,

$$\int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \sum_{n=-\infty}^{\infty} c_n \int_{-\infty}^{\infty} e^{jn\omega_0 t} e^{-j\omega t} dt \quad (5)$$

Using Fourier transformation, (5) can be rewritten as:

$$X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\omega_0) \quad (6)$$

where δ is Dirac delta function. $\delta(\omega - n\omega_0) = 0$, if $\omega \neq n\omega_0$; otherwise δ goes to infinity. Equations (3), (4), and (6) are used to obtain frequency spectrum, $X(\omega)$, of a given signal in time domain, $x(t)$. Spectrum of a periodic signal is a discrete spectrum with impulses at the frequencies: $\omega = n\omega_0$, $-\infty \leq n \leq \infty$ with corresponding areas of $2\pi c_n$. In this lab, we generate time domain signals (3) by using function generator; in order to obtain frequency domain signal (6), spectrum analyzer will be used.

II. Procedure

Function generator, oscilloscope, and spectrum analyzer.

III. PROCEDURE

1. Time and frequency domain representation of signals

- Set the function generator to the sinusoidal mode.
- Set the frequency to 200 kHz.
- Set the amplitude to 2Vpp.
- Display the signal on oscilloscope screen.
- Measure the amplitude and the period of the signal on oscilloscope (**time domain**).
- Connect the function generator to the spectrum analyzer.
- Measure the amplitude and the frequency of the signal on spectrum analyzer. (**frequency domain**).
- Change the frequency of the signal to lower to and higher frequencies and watch the variation of the signal by frequency on spectrum analyzer.