# Network Analysis I Laboratory EECS 70LA 

Spring 2018 Edition

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## Foreword

This manual was prepared for Network Analysis I laboratory. It consists of eight experiments. They were designed not only to help you understand simple network circuits but also to let you learn how to use the basic electronic instruments. You will have ample opportunities to build your own circuits, use your circuits to observe and study network characteristics, present your results, and comment on any discrepancy.

This manual has been revised and updated every year. We hope that you enjoy doing the experiments and gain lots of experimental skills and hand-on experience.

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March 29, 2014

## Summary

EECS 70LA Network Analysis I Laboratory
Objective: To enhance the understanding of electric circuits, network theorems, AC passive circuits, electronic measurement instruments.

Required for EE and $\mathrm{C}_{\mathrm{p}} \mathrm{E}$ Majors, one unit (design unit), effective spring 2014.
Laboratory Location: ET 202

## Experiments and Objectives

## 1. Ohm's Law and Applications (1 Week)

To study Ohm's law by measuring V-I characteristics of linear resistors and resistive components, such as light bulb; effects of internal resistance of measurement instruments on the measured resistance values.

## 2. Resistors in Series and Parallel, Wheatstone Bridge (1 Week)

To measure unknown resistance values by known resistors in Wheatstone bridge configuration; voltage divider circuits.

## 3. Circuit Analysis and Power Computation (1 Week)

To illustrate power conservation in the circuit using KCL and KVL; Y- $\Delta$ transform. Current measurement.

## 4. Nodal Analysis and Mesh Analysis (1 Week)

To verify mesh and nodal analyses of the circuits.

## 5. Network Theorems (1 Week)

To measure Thevenin voltage and resistance to represent a more general resistive (no L or C ) circuit with only measured equivalent parameters; to study optimum load resistance for maximum power transfer to the load; demonstration of source superposition theorem in linear circuits.

## 6. Time and Frequency Domain Representation of Signals and Amp. Modulation (1 Week)

To generate signals and their measurements in time and frequency domains; concept of modulation and demonstration amplitude modulation.

## 7. First Order R-L and R-C Circuits (1 Week)

To measure time constant of R-L and R-C circuits; dynamic responses of these circuits; measurements with oscilloscope.

## 8. Sinusoidal Steady State (2 Weeks)

To investigate and study AC circuits; current, voltage, phasor, and impedance measurement in RL, RC, and RLC circuits; resonance behavior in RLC circuits.

## Report Guidelines and Grading Policy

You are going to perform an experiment in a group of three students. Each group submits one lab report. Report is due one week after the experiment is performed. Attendance is mandatory. A report for a particular experiment will be accepted only when the experiment is actually performed. Before conducting the experiment, you are required to have read the experiment background and procedure from the lab manual and studied the related theory. You will also have prelabs or inclass questions for some of the labs, which help you have a better preparation for the experiments. Prelabs and in-class questions will be done individually. Prelabs must be submitted at the beginning of the lab session. Students might have makeup sessions if, and only if, they inform TA of accidents/other situations prior to class.

The grading of your laboratory report will focus on how well you achieve the class goals as documented by your writing. Your report should be brief but complete. Do not write sentences just to fill up space. Every word should have its importance, otherwise do not put it in the report. Your report will be judged not by its length but by its contents and quality.

The format to be followed in writing your report is:

- A cover page with the title of the experiment; your name, section number and student ID number; partner's name; and date of the experiment.
- A concise statement of the Objective (s) (1 paragraph) of the experiment in your own words. What do you expect to learn from the experiment? What are you going to measure/build/design?
- A Procedure section (1 or more paragraphs) which explain how you measured/built/designed, show the circuit schematics if necessary, includes a brief theoretical analysis if needed.
- A Results and Analysis section (1 or more paragraphs): This section will be the central part of your report. Be sure to include nominal values for Source (Vin), electrical components and instrument settings if relevant, and includes electrical measurements in tabular form to make them clear. Present your result as figures if needed, these figures will be the central result in your report. Compare the measured results with calculated ones. Explain any discrepancy.
For all the data measurement and data analysis, all axes must be labeled; No units, no credit; Any figure should be labeled (e.g. Fig. 1, Fig. 2, etc.); Any figure should be referenced in the main text in the order in which it appears.
It is fine to cut/paste figures, schematics from data sheets, lab manuals, or any other source. If you do this, you must cite the reference (e.g. data sheets, lab manual, etc.)
- A Discussion section (1 paragraph): Tell us what you think about the experiment, how well did it work?

The following is the breakdown of the lab grades.
Lab 1: 10/100 (Report 100\%)
Lab 2: 10/100 (Question in class 20\% + Report 80\%)
Lab 3: 15/100 (Prelab 20\% + Report 80\%)
Lab 4: 10/100 (Prelab 20\% + Report 80\%)
Lab 5: 10/100 (Question in class 20\% + Report 80\%)
Lab 6: 10/100 (Report 100\%)
Lab 7: 10/100 (Question in class 20\% + Report 80\%)
Lab 8: 15/100 (Report 100\%)
Pass-Fail exam: 10/100

## EECS 70LA <br> Introduction and Equipment Operations

1. OBJECTIVE: Introduction of EECS 70LA Laboratory contents, procedures, and regulations, and learning how to use all the equipment properly.

## 2. LABORATORY RULES

1. Do not bring newspapers or magazines to the laboratory.
2. No food and drink except bottle water.
3. No walkman and other audio gadgets.
4. Turn off your cellular phones.
5. Must wear shoes for your own protection.

## 3: LABORATORY PROCEDURES

1. Lab. TA will normally lecture for 20 minutes before each experiment.
2. Experiment time: 1 to 3 hours, must be finished in 2.5 hours.
3. Each group should check the equipment inventory on the station before experiment, and report any missing equipment to the TA.
4. Do not use any equipment without reading the manual, and without understanding the equipment functions.
5. TA will check the equipment of your station before you leave.
6. Each group needs to turn in a Lab Report in a week after the experiment.
7. You are required to read the description of next experiment in advance so that you come to the laboratory prepared.

## 4. SAFETY AND HAZARD GUIDELINES

Safety is always a major concern in the laboratory. Safety is more important than the experiments themselves. You should never compromise safety for the experiments.

## 5. GUIDELINES OF EQUIPMENT OPERATION

Students need to read the operation manuals to understand how to use the equipment to prevent damage to the equipment. Please take good care of them and do not abuse them. The equipment listed in the next section will be used very often in your experiments. You are required to read the operation manuals and to make sure that you know how to use the equipment properly. You also need to read the specifications of each equipment. The proper usage of the equipment will not only save your time but also extend the life of the equipment. If you are not sure about the key functions of the equipment, please read the manuals. Do not just try blindly. The operation of equipment and the understanding of equipment specifications is important engineering training that will eventually help you do a better job as an electrical and computer engineer.

## 6. EQUIPMENT LIST

Each station has its own equipment. You shall be responsible for the proper usage of all the equipment. The TA will verify the equipment list before you leave the laboratory.

## Description Qty. Manuals

| 1. Agilent 5034A Oscilloscope | 1 | User's guide |
| :--- | :--- | :--- |
| 2. Agilent 10073C Oscilloscope Probes | 2 |  |
| 3. Agilent 33250A Waveform generator | 1 | User's guide |
| 4. Agilent N9320B Spectrum analyzer | 1 | User's guide |
| 5. Agilent E3631A DC Power supply | 1 | User's guide |
| 6. Agilent 34401A Digital multimeter | 1 | User's and service guide |
| 7. Agilent 9320B Spectrum Analyzer | 1 |  |
| 8. Coaxial cables, BNC male to male | 2 |  |
| 9. Coaxial cables, BNC male to alligator clip 2 |  |  |
| 10. Banana to banana cables | 2 |  |
| 11. Banana to alligator clip cables | 2 |  |
| 12. Pliers, wire cutter | 1 set |  |
| 13. Breadboard | 1 |  |
| 14. Storage box | 1 |  |

## 7. EQUIPMENT OPERATIONS

TA will introduce you the equipments that will be used at the beginning of each lab section. The basic demonstrations, shown below, for these equipments will be performed in the first week of the lab:

1. Pick up two resistors with different values; read the value of each resistor using the color method. Measure the value of each resistor with the digital multimeter; and compare the measured value with the calculated one.
2. Use the digital multimeters to measure the voltage of the output of the DC power supply and verify the readings with that shown on the power supply meter. Pay attention to the specifications of input impedance in voltage mode and that in current mode. The input impedance affects the accuracy of your measurements but it is often ignored even by the experienced electrical engineers.
3. Display several kinds of waveforms generated by the pulse/function generator on the oscilloscope, e.g. sinusoidal waveform, square waveform, sawtooth, and pulses.

## EXPERIMENT 1: OHM'S LAW AND APPLICATIONS

Ohm's law and its applications are investigated in this experiment. The V-I characteristic of linear resistors is derived. Applications of Ohm's law include voltage and current division. Measurements of the equivalent resistance of a resistive arrangement are performed.

## I. BACKGROUND

## I. 1 Ohm's Law.

Ohm's law states that the voltage and current in a resistor are directly proportional. Resistors that obey the Ohm's law are called linear or ohmic resistors. In an ohmic resistor the ratio of the resistor voltage to the resistor current is independent of the voltage and current. This ratio is defined as the resistance of the resistor and it is measured in $\Omega$ (Ohms). Equation (1) expresses the Ohm's law. This equation is the terminal equation of a linear resistor.
$V=R I$
Equation (1) can also be written as in (2):
$I=G V$
Where $G=1 / R$ is the conductance of the resistor (measured in Siemens, S). Representation of the terminal equation of a resistor in the form of (1) is called resistance or impedance representation. Representation in the form of (2) is called conductance or admittance representation.

Resistors whose resistance varies with the voltage or the current in the resistor are called non-linear resistors. Non-linear resistors are described by a non-linear relation between their voltage and current.

## I. 2 The Voltage versus Current Characteristic of Linear Resistors.

The graph of the voltage v . the current of a linear resistor is a line called the V-I characteristic. With reference to Figure A, the V-I characteristic of the resistor always passes through the origin. Its slope is the resistance of the resistor. Its reciprocal slope is the conductance of the resistor. An ohmic resistor characteristic occupies only the first and third quadrant of the V-I plane. Thus, an ohmic resistor dissipates energy at any point of its characteristic.


Fig. A. The V-I characteristic of a linear resistor.
I. 3 Series and Parallel Combination of Resistors.

The equivalent resistance of resistors connected in series equals the summation of the resistance of each resistor. With reference to Figure B, the equivalent V-I characteristic of series resistors is obtained by the vertical summation of the V-I characteristics of the resistors. The slope of the equivalent characteristic is, thus, the summation of the slopes of each of the added characteristic.


Fig. B. The combined V-I characteristic of series resistors. (a) and (b) The V-I characteristics of the resistors. (c)
The combined V-I characteristic.
The voltage across the terminals of a series arrangement is distributed among the resistors proportionally to the resistance of each resistor (voltage division).
The equivalent conductance of resistors connected in parallel equals the summation of the conductance of each resistor. With reference to Figure C, the V-I characteristic of the parallel combination of resistors is obtained by the horizontal summation of the V-I characteristics of the resistors. The reciprocal slope of the V-I characteristic of parallel resistors is the summation of the reciprocal slopes of the V-I characteristics of the resistors.


Fig. C. The combined V-I characteristic of parallel resistors. (a) and (b) The V-I characteristics of the resistors. (c) The combined V-I characteristic.

Parallel resistors divide the current at the terminals of the arrangement proportionally to their conductance (current division).

## Measurement of Resistance Using an Ammeter and a Voltmeter.

INSTRUMENTATION: Agilent E3631A triple output DC Power supply, Agilent 34405A Digital multimeter.

PROCEDURE:
Use the resistor rated $10 \Omega$ in the arrangement shown in Figure 1a. Set the voltage of the supply to 1.5 V. Measure the voltage and the current indicated in the figure 1.

(a)

(b)

Fig. 1. The two arrangements of the voltmeter and ammeter for measuring resistance.
Repeat the procedure using the arrangement of Figure 1b. Tabulate your measurements as in Table 1.

|  | Circuit 1a | Circuit 1b |
| :---: | :---: | :---: |
| V |  |  |
| I |  |  |

Table 1 Measurement of $10 \Omega$ resistance using an voltmeter and an ammeter
Use the resistor rated $1 \mathrm{M} \Omega$ in the arrangement shown in Figure 1a. Set the voltage of the supply to 25 V. Measure the voltage and the current indicated in the figure 1. Repeat the procedure using the arrangement of Figure 1b. Tabulate your measurements as in Table 2.

|  | Circuit 1a | Circuit 1b |
| :---: | :---: | :---: |
| V |  |  |
| I |  |  |

Table 1 Measurement of $1 \mathrm{M} \Omega$ resistance using an voltmeter and an ammeter

## Derivation of I-V curve for different loads

INSTRUMENTATION: Agilent E3631A triple output DC Power supply.

## PROCEDURE:

For this part of the lab we will be building a very simple circuit consisting of the power supply and another component which, when connected to the power supply, draws power. We commonly call this component the 'load.' Since our power supply regulates the voltage, i. e. keeps the voltage constant across the load no matter what load we attach to it, the value of the resistance of the load will determine the current drawn from the power supply. You will be building this circuit using a resistor. Two other loads are available - a lamp, and a motor. Generate a plot of the current (I) flowing through the load vs. voltage (V) across the terminals of the load (I. e. the I-V curve).


## Simple resistance

A resistor has a very simple I-V curve. Since the voltage across a resistor is directly proportional to the current flowing through the device, according to a relationship known as Ohm's Law V=RI, we expect the I-V curve to be a straight line whose slope is related to the resistance. Lets see if that is correct. Build and test the characteristics of a circuit including a simple resistor and the power supply

- Connect the $100 \Omega$ resistor to the alligator clips inside one of the test boxes.
- Connect the outputs of the box to the power supply ( 6 V terminals) using the cables with banana plugs on both ends. This is your first real circuit. In making these connections we are following the diagram above with a $100 \Omega$ resistor as the load.

Now connect the $470 \Omega$ resistor and repeat the experiment and report the data in the table below

## Incandescent light bulb

Now use the incandescent Bulb as load, some loads look to the power supply like a simple resistor for a portion of their range of operation. The incandescent bulb used in this part of the lab is a resistive device whose resistance changes as the device heats up. Since the bulb is designed to heat up to a white hot state - this is how we get light - the resistance will vary as the filament in the bulb heats up. But once the bulb stays on for a while the resistance no longer change. For this range of voltage the behavior (until meltdown) can be modeled as a resistor. Create the voltage current table as in the case of the resistor for the light bulb and save your data. Make sure you have sufficient number of data point to catch the behavior.

PLOTTING I-V CURVES - Vary the voltage between 0 and 8V making note of the values of voltage and current for at least 10 values of the voltage. The power supply displays all the
information that you will need to fill in a table like the one below (alternately, you can enter the values into EXCEL or MATLAB). The power supply display shows both the voltage delivered to the terminals of the power supply and the amount of current being drawn from the power supply. Since we have a load connected directly to the supply these numbers are also the voltage across and current flowing through the load. Be sure to write the units next to the headings.

| Voltage | Current |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## REPORT:

## Resistance Measurements.

1. Compare and discuss the measurements from Figures 1 a and 1 b . What is in each case the $\%$ error between the measured and rated value of the resistor?
2. Why we obtain different resistor value in the two arrangements?
3. Why one arrangement seems to work at best for high value resistor and a second one for low value one?
4. Which configuration works best for high value and which one for low value resistor and why?

## I-V curves

Use the data you collect from the experiment to obtain the I-V curve of each load, plot the voltage recorded above along the x -axis against the current along the y -axis. You can use Excel or similar program to produce the plot. Label the axes, and give your graph a title.

1. What is the slope of the curve and how does this relate to the resistance of the load?
2. From the slopes of these I-V curves determine R
3. What is the difference in the I-V curve from the resistor versus the light bulb ?
4. Speculate how the temperature is affecting the apparent resistance (the slope of the curve at any point is the apparent conductance, which is the inverse of the resistance) using your I-V plot as evidence.
5. What is the relationship between the resistance of the light bulb and the brightness (or temperature)?

## EXPERIMENT 2: <br> RESISTORS IN SERIES AND PARALLEL, WHEATSTONE BRIDGE

## Direct Measurement of Resistance. The Wheatstone Bridge

Direct measurement of resistance is achieved by comparing the unknown resistance to a standard (known) resistance. Figure D shows the arrangement known as the Wheatstone bridge that is used for the direct measurement of resistance. In this arrangement, Rx is the unknown resistor and R3 is a variable resistor (rheostat).
The bridge consists of two voltage dividers formed by resistors R1 and R3 and by resistors R2 and Rx. The voltage across R3 and Rx are given by (3). The voltage Vo at the output terminals of the bridge is the difference between the two previous voltages and is given by

$$
\begin{align*}
& V_{3}=\frac{R_{3}}{R_{1}+R_{3}} \cdot V_{B}, V_{x}=\frac{R_{x}}{R_{x}+R_{2}} \cdot V_{B}  \tag{3}\\
& V_{o}=V_{3}-V_{x}=\left[\frac{R_{3}}{R_{3}+R_{1}}-\frac{R_{x}}{R_{x}+R_{2}}\right] \cdot V_{B} \tag{4}
\end{align*}
$$

The value of R3 varies by known steps until the voltage at the output of the bridge is zero. At this state the bridge is balanced. The condition to balance the bridge is derived from (4) and it is given in (5). The unknown resistance is given by (6).

$$
\begin{align*}
& \frac{R_{x}}{R_{2}}=\frac{R_{3}}{R_{1}}  \tag{5}\\
& R_{x}=\frac{R_{2} R_{3}}{R_{1}} \tag{6}
\end{align*}
$$



Fig. D. Arrangement of the Wheatstone bridge.
INSTRUMENTATION: Agilent E3631A triple output DC Power supply, Agilent 34405A Digital multimeter.

## PROCEDURE

## Wheatstone Bridge.

1) Construct the Wheatstone bridge in Figure D. Use a 9 V source and $\mathrm{R} 1=47 \mathrm{k} \Omega, \mathrm{R} 2=33 \mathrm{k} \Omega$, a resistance box and a third resistor for Rx. Measure the value of Rx using the ohmmeter.
2) Connect a voltmeter at the output of the bridge. Adjust the resistance box to balance the bridge Record the value of the box resistance.

Series and Parallel Combinations of Resistors.
Pick three resistors rated at $\mathrm{R} 1=1 \mathrm{k} \Omega, \mathrm{R} 2=2.2 \mathrm{k} \Omega$, and $\mathrm{R} 3=3.3 \mathrm{k} \Omega$. Measure their values in the Wheatstone-bridge.
Construct, one at a time, Arrangements a and b. Set the supply to 6 V .
For each arrangement, measure the indicated variables. Tabulate your results.

Arrangement a:


Fig. 2. Series resistors. Voltage division.
Table 2.
Measurements for Arrangement of Figure 2.

| V | I | $\mathrm{V}_{1}$ | $\mathrm{~V}_{2}$ | $\mathrm{~V}_{3}$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |

## Arrangement b :



Fig. 3. Parallel resistors. Current division.
Table 3.
Measurements for Arrangement of Figure 3.

| V | I | $\mathbf{I}_{1}$ | $\mathbf{I}_{2}$ | $\mathbf{I}_{3}$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |

REPORT:
Theoretical Development.
Provide the formulae for voltage and current division between two resistors.
Wheatstone Bridge.
Calculate from the measurements the value of resistor Rx. Compare it with its rated value.
Series Connected resistors.

- Use the measurements of Table 2 to verify the KVL in the circuit of Arrangement a.
- Compare these measurements with voltage division.
- Calculate from the measurements the power consumed by each resistor and the power delivered by the supply. Discuss your results.

Parallel Connected Resistors.

- Use the measurements of Table 3 to verify the KCL in the circuit of Arrangement b.
- Compare these measurements with current division.
- Calculate from the measurements the power consumed by each resistor and the power delivered by the supply. Discuss your results.


## EXPERIMENT 3: CIRCUIT ANALYSIS AND POWER COMPUTATION

The purpose of this experiment is to solve a circuit using Ohm's law, KCL or KVL, and verify power conservation in the circuit.

PRELAB
Apply the proper circuit transformation (Hint: $\Delta Y$ ) and solve the circuit in the figure below. Find the voltages, currents and power for each element of the simplified circuit (after you apply the transformation) including the voltage source of the circuit. Record and tabulate your data.


Figure 1
INSTRUMENTATION: Agilent E3631A triple output DC Power supply, Agilent 34405A Digital multimeter.

## PROCEDURE

A) Construct the circuit of Figure 1 using a breadboard, with the resistors values as indicated in the circuit. Set the power supply to 12 V and measure the voltages and current across each resistor. Next compute the power dissipate on each resistor as well from the generator and record these values.

Hint: To measure the current across each resistor you don't need to disconnect them from the circuit but you can simply measure the voltage across them, therefore since their resistance is known you can compute the current using Ohm's law this is much quicker and easy.
B) Replace the Delta configuration with the correspondent $Y$ configuration (note if you do not have the exact resistor value you can obtain by series 2 or more resistors, eg if you need 3 k as an example you can combine 1.2 k and 1.8 k in series). Measure the voltages and current across each resistor. Next compute the power dissipate on each resistor as well from the generator and record these values.

## REPORT:

Analyze the theoretical data and compare them with measured ones (from part B), provide explanation for any discrepancy if exist.
Show that the total power in the circuit is conserved (eg. Generated power is equal to dissipated one) within the accuracy of your instrumentation.
Compare based on your measure the total power dissipated by three resistors in delta configuration with the total power dissipated by the equivalent resistors in Y configuration.
Does the transformation from Delta to Y affect the voltage and current on R4? Explain why or why not.

## EXPERIMENT 4: <br> NODAL ANALYSIS AND MESH ANALYSIS

The purpose of this experiment is to solve a circuit using nodal network analysis and mesh or loop analysis.

PRELAB
Use nodal voltage analysis to set up the system of equations to solve the problem in Fig.1.


Fig. 1. The circuit for the nodal analysis.

Use loop analysis to set the proper system of equations to solve the circuit in Fig.2.


Fig. 2. The experimental setting for the mesh currents.

Use the following resistors:
$\mathrm{R}_{0}=560 \Omega, \mathrm{R}_{1}=1 \mathrm{k} \Omega, \mathrm{R}_{2}=6.8 \mathrm{k} \Omega, \mathrm{R}_{3}=3.3 \mathrm{k} \Omega, \mathrm{R}_{4}=4.7 \mathrm{k} \Omega, \mathrm{R}_{5}=1.8 \mathrm{k} \Omega, \mathrm{R}_{6}=10 \mathrm{k} \Omega$.

INSTRUMENTATION: Agilent E3631A triple output DC Power supply, Agilent 34405A Digital multimeter.

## PROCEDURE

A Verification of the nodal equations.
Construct the circuit of Figure 1 using a breadboard, with the resistors values as indicated in the circuit. Set the power supply to 6 V and measure the nodal voltages respect to ground.
$B$ Verification of the mesh equations.
Construct the circuit of Figure 2 using a breadboard, with the resistors values as indicated in the circuit. Set the power supply to 6 V and measure the current on each mesh.

## REPORT:

A. Theoretical Development.

1. Discuss the nodal and mesh method. When is one preferred over the other.
2. Solve the equations for both formulations (nodal and mesh) using MATLAB.
B. Verification of Nodal Equations.
3. Compare the measured values of the nodal voltages with the values from MATLAB
4. Write the nodal equations and verify using the measurements the KCL at each node.
C. Verification of Mesh Equations.
5. Compare the mesh currents measured with the values from MATLAB.
6. Write the circuit mesh equations and verify using the measurements the KVL in each mesh.

Analyze the theoretical data and compare them with measured ones, provide explanation for any discrepancy if exist.

## EXPERIMENT 5: <br> NETWORK THEOREMS

This experiment verifies some important network theorems: the Thévenin equivalent of a circuit, the maximum power transfer theorem, and the source superposition.

## I. BACKGROUND

I. 1 Representation of a Linear Resistive Circuit by a Thévenin Source.

A linear resistive circuit seen (observed) from two of its terminals (a two-terminal circuit) is equivalent to a Thévenin or a Norton source connected between the two terminals. These are called Thévenin or Norton equivalents respectively of the circuit.

Definitions: If a circuit contains only ohmic resistors, dependent, and independent sources (that is no capacitors or inductors) the circuit is called a linear resistive circuit.
The terminal voltage and current of a two-terminal linear resistive circuit are related by a linear equation. The Thévenin or Norton equivalency imply that a two-terminal linear resistive circuit can be replaced by a Thévenin or Norton source connected between the circuit terminals. Equivalency means that the circuit and the Thévenin source have identical V-I characteristics at their terminals.


Fig. A. Derivation of the Thévenin equivalent. (a) The circuit and its equivalent Thévenin source. (b) The short circuit measurement. (c) The terminal resistance measurement.

Figure A(a) shows a two-terminal linear-resistive circuit. When observed from its terminals the circuit appears as a Thévenin source with voltage VT (termed the Thévenin voltage) and resistance RT (termed the Thévenin resistance). The relation between the terminal voltage and current of the circuit is given by (1).
$V=-R_{T} I+V_{T}$
Figure B shows the terminal characteristic of (1). The intercept of this line on the V -axis is the Thévenin voltage. Figure A(a) shows that the Thévenin voltage can be measured directly across the open terminals of the network.


Fig. B. The V-I characteristic of the linear circuit.

The intercept of the circuit characteristic on the I-axis is the short circuit current (also equal to the Norton current). Figure $A(b)$ shows that this current is measured when we short the terminals of the circuit. The short circuit current is given by (2).

$$
\begin{equation*}
I_{s c}=\frac{V_{T}}{R_{T}} \tag{2}
\end{equation*}
$$

The slope of the characteristic is the opposite of the Thévenin resistance, RT. The Thévenin resistance can be measured either (a) from the ratio of the Thévenin voltage to the short circuit (Norton) current:

$$
\begin{equation*}
R_{T}=\frac{V_{T}}{I_{s c}} \tag{3}
\end{equation*}
$$

Or (b) with an ohmmeter across the circuit terminals, as shown in Figure $A(c)$ : all independent sources of the circuit are made zero (voltage sources are replaced by a short circuit, current sources by an open circuit). The resistance measured at the terminals is, then, the Thévenin resistance.
The experimental determination of the Thévenin equivalent of a circuit requires, therefore, these measurements: (a) Open circuit voltage (Figure $A(a)$ ); and one of the following: (b) short circuit current (Figure $\mathrm{A}(\mathrm{b})$ ), or (c) terminal resistance (Figure $\mathrm{A}(\mathrm{c})$ ).

## PROCEDURE

## A. Experimental Determination of the Thévenin Equivalent

1. Construct the circuit shown in Figure 1 , where $\mathrm{R} 1=1.2 \mathrm{k} \Omega, \mathrm{R} 2=2.4 \mathrm{k} \Omega$, and $\mathrm{R} 3=4.7 \mathrm{k} \Omega$. Set the supply at 8 V . Perform the following measurements:
a. Measure the voltage across the open terminals a-b.
b. Short the terminals a-b through an ammeter. Measure the current.
c. Disconnect the supply from the circuit and replace it by a short circuit. Measure the resistance of Terminals a-b using an ohmmeter.


Fig. 1. Experimental arrangement.
B. Maximum Power Transfer.
2. Connect a variable resistor between the terminals a-b of the previous circuit. Vary its value in the range from 1.0 to $10 \mathrm{k} \Omega$ by increments of $1000 \Omega$. For each value of the resistance, measure the voltage, V, across it. Use Table 1.

Table 1.
Measurements for the maximum power transfer.

| $\mathbf{R}$ |  |  |  |  |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{V}$ |  |  |  |  |  |  |  |  |  |  |

## C. Source Superposition.

3. Construct the circuit of Figure 2 by connecting a second source across the terminals a-b of the previous circuit. Use R4=5.1 $\mathrm{k} \Omega$. Use the two outputs of the power supply for each of the sources.


Fig. 2. The experimental arrangement with a Thevenin source.
4. Set V1 to 8 V and V 2 to 6 V . Measure V and I .
5. Replace V2 by a short-circuit. Measure V and I.
6. Reconnect V2 and set its value to 6 V. Replace V1 by a short-circuit.. Measure V and I.

## REPORT

A. Theoretical Development.

1. (a) Discuss the Thévenin and Norton equivalents and source transformation. Give an example of a twoterminal circuit for which no-equivalent Thévenin source exists.
(b) Prove the theorem of maximum power transfer.

## B. Determination of Thévenin equivalent.

2. Calculate the Thévenin equivalent of Figure 1 from:
(a) The measurements in Procedures 1 a and 1 b .
(b) The measurements in Procedures 1a and 1c.

Compare with the theoretical Thévenin equivalent.
C. Maximum Power Transfer.
3. Plot the power dissipated on the variable resistor vs the resistance in Procedure 2. Tabulate your calculations. At what value of the resistor does maximum power occur? How much is the maximum power? How close do these values correspond with the theoretical ones from the maximum power transfer theorem?
D. Source Superposition.
4. Do the measurements of the voltage and current from Procedures 4,5 , and 6 agree with the principle of source superposition? Explain.

## EXPERIMENT 7: <br> FIRST ORDER R-L AND R-C CIRCUITS

## Objectives:

The objective of this experiment is to observe the response of the first order R-C and R-L circuits. The experiment demonstrates a method for measuring the time constant.

## I. BACKGROUND

I. 1 Measurement of the Natural Response of First Order Circuits.

The natural response of an R-C circuit, shown in Figure A, is given by (1).

$$
\begin{equation*}
v(t)=v_{0} \cdot e^{-\frac{t}{t}} \tag{1}
\end{equation*}
$$

Where, vo is the value of the capacitor voltage v at $\mathrm{t}=0 \mathrm{~s}, \tau=\mathrm{RC}$ is the time constant of the circuit. The time constant gives the rate at which the voltage decays to zero. In circuits, this decay response is due to ohmic losses.


The time constant can be measured by one of the following graphical methods.
(a) The Tangential Line. A line tangential at a certain point of the response curve is drawn. The line intersects the time axis exactly in one time constant from the point of tangent. Figure B shows the graph of the response from (1). Note that the response begins $a t=0.05 \mathrm{~s}$. With reference to Figure B, a line is drawn tangential to the curve at $\mathrm{t}=0.05 \mathrm{~s}$ (that is at the onset of the decay). The line is extended until it intersects the time axis. This occurs at $\mathrm{t}=0.25 \mathrm{~s}$. Therefore, the time constant of the response is

$$
\begin{equation*}
\tau=0.25-0.05=0.2 \mathrm{~s} \tag{2}
\end{equation*}
$$

This method is suitable if a hard copy of the response graph is available.


Fig. B. Measurement of the time constant of an exponentially decaying response.
(b) The $63.22 \%$ Decay. The second method is suitable for measurements on the oscilloscope. From (1), every time interval equal to one time constant the response decays by $63.22 \%$. Equivalently, at the end of every interval equal to one time constant, the response is at the $100-63.22=36.78 \%$ of its value at the beginning of the interval. This method of measurement is demonstrated in Figure B. The response begins its decay at $\mathrm{t}=0.05 \mathrm{~s}$, at that point the value of the curve is 1 . The response reaches $36.78 \%$ of its initial value at $\mathrm{t}=0.25 \mathrm{~s}$. Thus the time constant is

$$
\begin{equation*}
\mathrm{t}=0.25-0.05=0.2 \mathrm{~s} \tag{3}
\end{equation*}
$$

The results of (3) and (2) are in agreement.

## I. 2 The Step Response of First Order Circuits.

The step response of the first order circuit in Figure A is given by (4).

$$
\begin{equation*}
v(t)=v_{f}\left(1-e^{\frac{t}{\tau}}\right) \tag{4}
\end{equation*}
$$

Where, $\mathrm{v}_{\mathrm{f}}$ is the final value of the response (also, called the constant steady state of the response). The step response of the circuit is characterized by the same time constant as the natural response. The time constant affects the step response in a similar manner as the natural response. Therefore, the methods discussed previously apply in this case, as well.
On the oscilloscope the time constant of the step response can be measured by measuring the time the output requires to reach $63.22 \%$ of its final value.

The natural and step responses of a first order R-L circuit, shown in Figure C, are the same as in (1) and (4) given for the circuit current $i$. The circuit time constant is $\tau=\mathrm{L} / \mathrm{R}$.


Fig. C. The experimental R-L circuit.

## I. 3 Oscilloscope Observations.

Non-storage oscilloscopes must display periodic wave forms, so that measurements and observations can be easily made. The natural response of a circuit can be observed along with its step response, if a square-pulse periodic excitation is applied to the circuit.
Figure D shows the response of a first order circuit to a periodic square pulse. The first part of the circuit response is the step response described by (4). The second part of the circuit response, when the excitation becomes zero, is the natural response described by (1). If the response is allowed to decay sufficiently, the following period will begin with another step response followed by the natural response. Thus, the circuit response will alternate between step and natural responses. To achieve good results, therefore, the period of the excitation must be adjusted to 7 to 10 times the time constant of the circuit.


Fig. D. Circuit response to a square pulse excitation with large period. (a) The excitation. (b) The circuit response.

## II. INSTRUMENTATION

Function generator, Oscilloscope,

## III. PROCEDURE:

Become familiar with the signal generator and the oscilloscope.

- The R-C Circuit.


Fig. 1. The experimental R-C circuit.

1. (a) Construct the R-C circuit of Figure 1. Use a $510 \Omega$.
(b) Apply the square pulse generator to produce a periodic response of the circuit that appears clearly on the oscilloscope. The period of the source is adjusted to be at least 20 times higher than the calculated time constant of the circuit. This is necessary, so that the capacitor reaches steady state, in both charging and discharging intervals.
(c) Observe on the oscilloscope the quantities shown in the figure. Measure the circuit time constant by the $63 \%$ decay method described above.
(d) Plot on a millimeter paper one period of the voltage and current response. Use appropriate scale. Label the axes. Transfer measurements from the oscilloscope to the paper by inspection.
2. Change the resistance in Figure 3 to $250 \Omega$. Repeat Procedure 1. Use the same millimeter paper and the same scale. Align the graphs. Do not change the setting of the source.
B. The R-L Circuit.
** Consider internal resistance of the inductor, which is $235 \Omega$, to calculate the time constant.
3. (a) Construct the circuit of Figure 2. Set the resistance box to $510 \Omega$.
(b) Apply a square pulse voltage. Set the frequency of the source to correspond to 10 times the time constant of the circuit.
(c) Repeat the same step 1c for this circuit.
4. Change the resistance to $250 \Omega$. Perform again step 1c.
5. Simulate the experiment circuit (Figure 2) in Pspice. For each value of R used above compute the circuit time constant.


Fig. 2. The experimental R-L circuit.

## IV. REPORT

A. Theoretical Development.

1. In theory discuss the natural and step responses of R-C and R-L circuits.
2. Define the duty cycle of a pulse. What is the duty cycle of a square pulse?
3. What is the difference in the setting of x 1 and x 10 of the oscilloscope probes?
4. How do we adjust the voltage and time scale of the oscilloscope? What is the purpose of calibrating a channel?
5. What is a waveform? What kind of waveforms are best observed and measured by oscilloscopes similar to the one used in the experiment?
6. What is a current measuring resistor? What was the value of the current measuring resistor used in the experiment? Is this value satisfactory?
B. The $R$-C Circuit.
7. (a) Explain the response observed in Procedure 1. Indicate which interval represents the natural and which interval represents the step response of the circuit.
(b) What is the circuit time constant measured for each value of R ?

## EXPERIMENT 8: <br> SINUSOIDAL STEADY STATE

This experiment demonstrates the properties of ac networks. The concept of impedance is discussed. Phasors are demonstrated through oscillograms.

## I. BACKGROUND

I. 1 AC Measurements. The RMS Value.

AC instruments measure the rms (root mean square) value of the ac voltage and current. For a periodic wave form of period T , the rms value is given by (1):

$$
\begin{equation*}
X_{m m s}=\sqrt{\frac{1}{T} \int_{0}^{T} x^{2}(t) \cdot d t} \tag{1}
\end{equation*}
$$

The rms of a sinusoidal quantity is given by (2):

$$
\begin{equation*}
X_{m s}=\frac{X_{m}}{\sqrt{2}} \tag{2}
\end{equation*}
$$

Where, Xm is the amplitude of the sinusoidal quantity. The distinction between dc and ac instrumentation is important. In circuits operating in sinusoidal steady state, a dc instrument will consistently indicate zero voltage and current.
The rms value of a current or voltage is, also, referred to as its effective value. This shows an equivalency between ac and dc quantities in the following sense:
An ac current (voltage) produces the same power dissipation on a resistor as a dc current (voltage), which has an average (dc) value equal to the rms (effective) value of the ac current (voltage). Thus, an ac current of amplitude 1.41 A is equivalent, in the previous sense, with a dc current of average value equal 1 A .
In practice, different notation is used to distinguish rms and average measurements. Thus, 10 V AC means that the rms value of the voltage measured is 10 V . While, 15 V DC means that the average value of the voltage measured is 15 V . Additional notation is available for amplitude description of ac wave forms. Thus, 20 mA p-p (red 20 mA peak-to-peak) means that the current varies by 20 mA between two successive peaks. For a sinusoidal current this means that its amplitude is 10 mA .
I. 2 Measurements of Phasors and Impedance in the Sinusoidal Steady State.

The currents and voltages of a circuit that operates in sinusoidal steady state are represented by phasors. With reference to Figure A, the impedance expresses the relation between the phasor of the terminal voltage and the phasor of the terminal current of a two-terminal RLC combination.
Conversely, every passive element or two-terminal combination of passive elements possesses an impedance, which expresses the Ohm's law as follows:


Fig. A. Phasors and impedance in sinusoidal steady state.
The magnitude of the phasor of a quantity can be measured by measuring the rms value of the quantity. The phase of the phasor can be measured using the oscilloscope. Figure B shows the oscillogram of the current and voltage at the terminals of an RLC arrangement. It is important to indicate or note the scale of the oscillogram. Notice that the oscilloscope probes measure voltage (unless a current probe is available). Appropriate scaling is required to obtain the correspondence between the oscilloscope vertical divisions and the measured current.


Fig. B. The oscillogram of voltage and current.

For the example of Figure B, the period of the oscillations is, by reading the scale, (T/2)=4 divisions $x 2 \mathrm{~ms} /$ division $=8 \mathrm{~ms}$. Therefore, the period is 16 ms . This corresponds to a frequency of 62.5 Hz .
The amplitude of the voltage oscillation on the oscilloscope is, by reading the scale from the reference line, 2.5 divisions. Therefore, the amplitude of the voltage is $2.5 \mathrm{divx} 10 \mathrm{~V} / \mathrm{div}=25 \mathrm{~V}$. The rms value of the voltage is, therefore, 17.68 V .
Likewise, by reading the scale, the current oscillation on the oscilloscope has an amplitude of 1.8 divisions and the actual current has an amplitude of 0.9 A . Its rms value is 0.636 A .

The zero crossing displacement between voltage and current is $\mathrm{d}=0.8 \mathrm{div} \times 2 \mathrm{~ms} / \mathrm{div}=1.6 \mathrm{~ms}$. This corresponds to a phase difference of $\Delta \varphi=360^{\circ} \times 62.5 \mathrm{~Hz} \times 1.6 \mathrm{~ms}=36^{\circ}$, with the current zero crossing leading the voltage zero crossing.
From this measurements, the phasors of the voltage and current and the impedance of the RLC arrangement can be calculated. Assuming the voltage as reference:

$$
\begin{align*}
& \bar{V}=25^{\angle 0^{\circ}} \mathrm{V} \\
& \bar{I}=0.9^{\angle 36^{\circ}} \mathrm{A} \\
& |\bar{z}|=\frac{|\bar{V}|}{|\bar{I}|}=\frac{25}{0.9}=27.78 \Omega  \tag{4}\\
& \angle \bar{z}=\angle \bar{V}-\angle \bar{I}=0^{\circ}-36^{\circ}=-36^{\circ}
\end{align*}
$$

Note that since the current is leading the voltage, the current's phase angle with respect to the voltage is positive. The impedance angle is the angle between current and voltage. Hence, the calculation of (4) results in negative impedance angle. This means the RLC arrangement is capacitive.

Its equivalent resistance is: $\mathrm{R}=|\mathrm{Z}| \cos (-\Delta \varphi)=22.47 \Omega$.
Its equivalent reactance is: $\mathrm{X}=|\mathrm{Z}| \sin (-\Delta \varphi)=-16.33 \Omega$, or $16.33 \Omega$ Capacitive.

## II. INSTRUMENTATION

Function generator, Oscilloscope.

## III. PROCEDURE:

A. Measurements of Capacitive Impedance.

1. Construct the circuit of Figure 1. Set the signal generator to the sinusoidal mode. Set the frequency to 1 K Hz and the amplitude to 10 Vpp .
2. Display on the oscilloscope the current and voltage of the capacitor.
3. Vary R from $50 \Omega$ to $500 \Omega$ in increments of $50 \Omega$. For each value of R measure the amplitude of the voltage and current. With the voltage as reference, also measure the phase of the current. Use Table 1.


Fig. 1. The experimental RC circuit. $\mathrm{C}=1 \mu \mathrm{~F}$.

| R |  |  |  |  |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\|\mathrm{V}\|$ |  |  |  |  |  |  |  |  |  |  |
| $\mathrm{I} \mid$ |  |  |  |  |  |  |  |  |  |  |
| $\varnothing \mathbf{I}$ |  |  |  |  |  |  |  |  |  |  |

B. Measurement of Inductive Impedance.
4. (a) Construct the circuit of Figure 2. Set the source to the same values as in the previous circuit. (b) Vary R from $50 \Omega$ to $500 \Omega$ in increments of $50 \Omega$. For each value of R measure the amplitude of the voltage, the amplitude of the current, and the phase of the current to the voltage. Use Table 2.


Fig. 2. The experimental RL circuit. $\mathrm{L}=10 \mathrm{mH}$.
Table 2.
Impedance measurements in the RL circuit.

| R |  |  |  |  |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\|\mathbf{V}\|$ |  |  |  |  |  |  |  |  |  |  |
| $\|\mathbf{I}\|$ |  |  |  |  |  |  |  |  |  |  |
| $\varnothing \mathbf{I}$ |  |  |  |  |  |  |  |  |  |  |

C. Measurement of the Impedance of an RLC Arrangement.
5. Construct the circuit of Figure 3. Maintain the settings of the source. Measure the magnitude of the supply voltage. Measure the magnitude and phase (with respect to the supply voltage) of the inductor current, the capacitor current and the supply current.


Fig. 3. The experimental parallel RLC circuit. $\mathrm{R}=100 \Omega, \mathrm{~L}=10 \mathrm{mH}, \mathrm{C}=1 \mu \mathrm{~F}$.
D. Compensation of Source Reactance.
6. (a) Construct the circuit of Figure 4 with the capacitor disconnected. Maintain the source at the settings of the previous procedure. Measure the phasors of the supply voltage and current and the phasor of the voltage across the resistor. Use the voltage across the resistor as reference.
(b) Connect the capacitor. Repeat the previous measurements. Also, measure the phasor of the capacitor current.


Fig. 4. The experimental circuit to observe compensation. $\mathrm{R}=100 \Omega, \mathrm{~L}=10 \mathrm{mH}, \mathrm{C}=1 \mu \mathrm{~F}$.

## E. Measurement of Parallel Resonance.

7. Construct the circuit of Figure 5. Set the supply to 1 V in the sinusoidal mode.
8. Vary slowly the frequency of the source until the amplitude of the supply current becomes minimum. For this point record the phasors of the supply voltage (reference), supply current, inductor current, and capacitor current.
9. Take the same measurements as in 8 for five frequencies below the frequency you recorded in 8 .
10. Repeat 9 for five frequencies above the frequency in 8.

For your measurements in 8, 9 and 10, use Table 3.


Fig. 5. The experimental setting to record resonance. $\mathrm{R}=100 \Omega, \mathrm{~L}=10 \mathrm{mH}, \mathrm{C}=1 \mu \mathrm{~F}$.
Table 3.
Measurements of resonance.

| f |  |  |  | - | - | - | - | - | - | - |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\|\mathrm{V}\|$ |  |  |  |  |  |  |  |  |  |  |  |
| [IL |  |  |  |  |  |  |  |  |  |  |  |
| ØIL |  |  |  |  |  |  |  |  |  |  |  |
| $\underline{\mathrm{IC}}$ |  |  |  |  |  |  |  |  |  |  |  |
| ØIC |  |  |  |  |  |  |  |  |  |  |  |
| \|Is |  |  |  |  |  |  |  |  |  |  |  |
| ØIs |  |  |  |  |  |  |  |  |  |  |  |

## IV. REPORT:

## A. Theoretical Development.

1. In theory explain the meaning of impedance. Explain the basic power calculations in ac circuits.
B. Measurement of RC and RL Impedance's.
2. For each of the circuits in Figures 1 and 2 and using Tables 1 and 2, respectively, perform the following calculations.
(a) Calculate the impedance of the circuit using the measurements.
(b) Calculate the theoretical value of the impedance of the circuit for the values of R in Tables 1 and 2. Tabulate magnitude and angle.
(c) Plot vs R on millimeter paper the impedance magnitude from a and b . Use different axes on the same paper and plot vs $R$ the impedance angle from $a$ and $b$. Use one graph paper for Circuit 1 and another graph paper for Circuit 2. Compare the experimental with the theoretical graphs.
(d) How does the behavior of each circuit change as R increases?
C. Phasor Diagrams. Verification of KVL and KCL.
3. (a) Draw on millimeter paper and on scale the phasor diagram of Circuit 3 using your measurements. Compare with theoretical values.
(b) Analyze the supply current to its active and reactive components using the supply voltage as reference. Compare with your measurements. What is the real and reactive power of the supply? What is the reactive power of the inductor and capacitor?
D. Phasor Diagrams. Application to Reactance Compensation.
4. Draw the phasor diagram of Circuit 4 with and without the capacitor. Use your measurements. Explain with appropriate theory the difference of these diagrams. What is the active and reactive power of the source?
E. Parallel Resonance.
5. (a) For the circuit of Figure 5, draw on separate graphs the magnitude and angle of the circuit impedance vs the frequency of the source. Use your measurements. Mark the resonance frequency on the graph. Compare with the theoretical value. What is the magnitude and angle of the circuit impedance at resonance? In what frequency range is the impedance inductive? In what frequency range is the impedance capacitive?
(b) At the resonance frequency, draw the phasor diagram of the circuit. Discuss it. What is the reactive power of the capacitor and inductor? Compare the supply current with the resistor current. Compare the inductor and capacitor currents.
(c) Draw the locus of the phasor of the supply current for different frequencies. In what frequency range is the current leading the supply voltage? In what frequency range is the current lagging the supply voltage?
